Uncertainties Related to Thermal Expansion in Dimensional Metrology

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Abstract: Thermal expansion effects are very important in dimensional metrology. In this paper a measurement model, and associated equations, are developed for the case of a one-dimensional measurement of a steel test gage using a measuring machine and master gage. After presenting the uncertainty components for this measurement, several example measurement situations with difference levels of temperature control are calculated and discussed. For each situation, the magnitude of the different sources of uncertainty are compared in order to rationally allocate resources to improve the overall measurement uncertainty.

1. Introduction

In dimensional measurement the uncertainty is often dominated by the effects of thermal expansion. [1] In this paper I discuss these effects, their sources, and the methods used to determine the uncertainty components. In an extended example, the thermal uncertainty components for the measurement of a steel gage on a one-dimensional universal length measuring machine (ULMM) is derived for different levels of laboratory temperature control and measurement. By increasing the knowledge of the temperature of the instrument and gages, the uncertainty of the measurement is dramatically lowered.

2. The Model

In order to make a length measurement, we must take the actual measured values and calculate what the length would be at exactly 20 ºC (68 ºF). In the most general case we will have a measuring machine with a scale (S), a master gage (M), and a test gage (G). To make the corrections we must have the temperature and coefficients of thermal expansion (CTEs) of each.

The actual measurement equation is:

$$S(t_s) = G(t_g) - M(t_m), \tag{1}$$

where M is the length of the master gage, G is the length of the test gage, and S is the apparent difference in the scale readings for M and G. Each of these depends on temperature. If we denote each CTE with α, and the scale is calibrated to be correct at 20 ºC, we find that:

$$M(t_m) = [1+\alpha_m(t_m - 20)]\ M_{20} \tag{2}$$

and

$$G(t_g) = [1 + \alpha_g(t_g - 20)]\ G_{20}\ . \tag{3}$$

We also have the scale readout, $S(t_s)$. What we would like, of course, is the actual difference in length between $M(t_m)$ and $G(t_g)$. However, the scale reading is not correct because the scale also changes with temperature.

Suppose the temperature is above 20 ºC. The scale is now longer, and the distance we measure will seem smaller than it really is. We thus have to correct the scale reading by enlarging it in proportion to the thermal expansion. Thus, the actual length difference between the gage and master is S_{meas},

$$S_{meas} = [1+ \alpha_s(t_s - 20)]\, S_{20}\,. \tag{4}$$

Putting these together, we get:

$$[1+ \alpha_s(t_s - 20)]\, S_{20} = [1 + \alpha_g(t_g - 20)]\, G_{20} - [1+\alpha_m(t_m - 20)]\, M_{20}\,. \tag{5}$$

Now we make a small replacement to make the equation easier to handle; we use the Greek Θ to stand for (t − 20). We then get:

$$(1+\alpha_s\Theta_s)\, S_{20} = (1+\alpha_g\Theta_g)\, G_{20} - (1+\alpha_m\Theta_m)\, M_{20}\,. \tag{6}$$

If we solve for the length of the gage, G_{20}, we get:

$$G_{20} = \frac{(1+\alpha_m\Theta_m)M_{20} + (1+\alpha_s\Theta_s)S_{20}}{(1+\alpha_g\Theta_g)} \tag{7}$$

Now, we can simplify this a bit by noting that the second term in the denominator is much smaller than 1. We can expand $(1 + \alpha_g\Theta_g)^{-1}$ in a Taylor series, and keep only the first two terms,

$$(1 + \alpha_g\Theta_g)^{-1} \approx (1 - \alpha_g\Theta_g)\,. \tag{8}$$

Then we get as our equation:

$$G_{20} \approx (1 + \alpha_s\Theta_s)(1 - \alpha_g\Theta_g)\, S_{20} + (1 + \alpha_m\Theta_m)(1 - \alpha_g\Theta_g)\, M_{20}\,. \tag{9}$$

Again, since all the αΘ terms are much smaller than 1, we can multiply out the right hand side and ignore all of the terms of order $(\alpha\Theta)^2$ and higher. We are left with the final equation for the length of the gage:

$$G_{20} \approx (1 + \alpha_s\Theta_s - \alpha_g\Theta_g)\, S_{20} + (1 + \alpha_m\Theta_m - \alpha_g\Theta_g)\, M_{20}\,. \tag{10}$$

How good is this equation? Let us take an extreme case of a measurement at 0 ºC on plastic. Plastics have very large CTEs, some nearly $100\text{x}10^{-6}$ /ºC. Thus, the term

$$\alpha\Theta \approx 100 \text{ x } 10^{-6} \text{ /ºC x } 20 \text{ ºC x } L \approx 0.002\, L. \tag{11}$$

The second order terms, being $(\alpha\Theta)^2 \approx 0.000\ 004$ L, are about 500 times smaller. For most laboratory conditions, the ratio is much larger, and therefore these second order terms are negligible.

Thus, to make thermal corrections to a general measurement, we need a number of quantities:

α_s	CTE of the scale of the measurement instrument
α_m	CTE of the master gage
α_g	CTE of the test gage
t_s	temperature of the scale
t_m	temperature of the master gage
t_g	temperature of the test gage
M_{20}	calibrated length of the master gage
$S(t_s)$	scale reading of the measurement instrument

All of these quantities have some uncertainty, u, associated with them. The thermal error terms are:

Uncertainty in CTE of the scale	$u(\alpha_s)\ \Theta_s\ S(t_s)$
Uncertainty in CTE of the test gage	$u(\alpha_g)\ \Theta_g\ [S(t_s) + M_{20}]$
Uncertainty in CTE of the master gage	$u(\alpha_m)\ \Theta_m\ M_{20}$
Uncertainty in scale temperature	$u(\Theta_s)\ \alpha_s\ S(t_s)$
Uncertainty in test gage temperature	$u(\Theta_g)\ \alpha_g\ [S(t_s) + M_{20}]$
Uncertainty in master gage temperature	$u(\Theta_m)\ \alpha_m\ M_{20}$

There are also the uncertainties of the scale reading and the length of the master gage to include in the overall uncertainty budget. Since we are focusing on sources associated with thermal expansion, however, we will not say much about these. Usually the scale reading uncertainty comes from the certificate or the manufacturer's specification if it is certified to its specification, rather than being calibrated. If the correction from the master gage calibration certificate is used, the scale reading uncertainty is taken from the calibration laboratory's stated uncertainty. If the calibration only certifies the gage to an accuracy class or grade, the width of the class or grade tolerance is taken as a rectangular distribution. [2]

The rest of the quantities are more difficult to estimate; you actually have to think a bit. In some cases the CTEs can be determined to some precision from calibration. At NIST we have calibrations of the CTEs of our gages, either in house or from calibration reports from the manufacturer. In these cases the uncertainty in the CTE can be quite small, $0.1\text{x}10^{-6}$ /ºC or better. Without such detailed information you must use whatever information you can get. The range of CTE for "steel" is quite large, but there are only a few gage block steels, and the range of CTE for these is somewhat smaller. The Standards for gage blocks [3] usually require that the CTE of a steel gage block is $11.5\text{x}10^{-6}$ /ºC with a tolerance of $\pm\ 1\text{x}10^{-6}$ /ºC. Other materials, such as tungsten carbide, chrome carbide, ceramic, etc. have no specification and most people will assume ± 10 % as the uncertainty. [4]

The temperatures are more complicated, still. Some notes:

1. If you only have one thermometer, and it is used only to monitor the room, then you must use the daily variation in the room temperature as the uncertainty for everything. This is a very large number, but uncertainty is a measure of your ignorance of the measurement and if you don’t measure something you are pretty ignorant.

2. If your thermometer is calibrated you still cannot automatically use the uncertainty on the certificate as your uncertainty. Many thermometers, particularly low cost thermistors, drift over their calibration cycle. I have been in many labs that have thermometers with certificates that say the uncertainty in the thermometer calibration is 0.01 ºC, but when I examine the calibration history I find that the thermometer is adjusted by 0.05 ºC to 0.10 ºC or more each time it is recalibrated. In general, the historical variation in the thermometer is the acceptable uncertainty.

3. As in most “meets manufacturer’s specification” types of calibrations, if the thermometer is not adjusted (see note 2 above), you can take the specification as a rectangular distribution.

To demonstrate the effects of thermally related sources of uncertainty, the uncertainty budget for a single measurement is analyzed. The first example if for a lab with only the most basic knowledge of the environment, and succeeding examples illustrate how the uncertainty can be lowered by changing the level of temperature measurement and control. The example is for the comparison of a test ring gage to a master ring gage using a universal length measuring machine (ULMM). [5] The master ring gage is calibrated by an accredited lab with an uncertainty of 0.5 µm.

3. Example 1: 100 mm customer ring gage calibrated using a 100 mm master ring on a long range ULMM. Lab has one thermometer to monitor room.

The typical uncertainty budget for this measurement equation looks like:

Master Gage	100 mm ring gage	uncertainty 500 nm (k=2)
ULMM spec	0.2 µm + 0.5 x 10^{-6} L	"accuracy specification"
Test Gage Material: steel	CTE = 12 x 10^{-6} /ºC	uncertainty 10 %
Master Material: steel	CTE = 12 x 10^{-6} /ºC	uncertainty 10 %
Scale Material: glass	CTE = 7 x 10^{-6} /ºC	uncertainty 10 %

In addition:

- Room temperature variation is ± 1 ºC.

Since we only have one thermometer and it determines some sort of room average, we are hard pressed to say we know the temperature of anything to better than 1 ºC. So, we will take the uncertainty in all of the temperatures as a rectangular distribution of ± 1 ºC. Note that if we were to measure the temperatures with the single thermometer, the uncertainties would be correlated and the analysis would be more complicated. Here, however, we are not using the thermometer to measure the actual temperatures of the scale and gages, just to set limits on their variations.

We also have to estimate the value of the temperature difference between the scale and gages and 20 °C. In a typical room the temperature change is roughly linear, so the average difference between 20 °C and the scale and gages is 0.5 °C. It can be argued that this is an over estimate because the gages and ULMM act like low pass filters on the room air temperature, so that the variations are more like sine waves than saw-tooth waves. This calculation is too complex for most labs and the changes are small compared to other sources of uncertainty. We will use 0.5 °C.

Source	Uncertainty / Range	Dist.	Std. Unc. of Factor	Sensitivity Coeff.	Standard Uncertainty	Std Unc. (µm)
Test Gage Temp. (steel)	1 ºC	rect	0.58 ºC	12 x 10^{-6} L/ºC	6.9x10^{-6} L	0.69
Master Gage Temp. (steel)	1 ºC	rect	0.58 ºC	12 x 10^{-6} L/ºC	6.9x10^{-6} L	0.69
Scale Temp. (glass)	1 ºC	rect	0.58 ºC	7 x 10^{-6} L/ºC	4.0x10^{-6} L	0.40
CTE (scale)	0.7x10^{-6} /ºC	rect	0.40x10^{-6} /ºC	0.5 ºC	0.20x10^{-6} L	0.020
CTE (master gage)	1.2x10^{-6} /ºC	rect	0.7x10^{-6} /ºC	0.5 ºC	0.35x10^{-6} L	0.035

CTE (test gage)	$1.2x10^{-6}$ /ºC	rect	$0.7x10^{-6}$ /ºC	0.5 ºC	$0.35x10^{-6}$ L	0.035
Length of Master Gage	0.50 µm	norm	0.250 µm	1	0.250 µm	0.25
Scale Specification	0.25 µm	rect	0.150 µm	1	0.150 µm	0.15
				Combined Standard Uncertainty		1.10
				Expanded Uncertainty (k=2)		2.20

Let's examine the biggest parts of this uncertainty. If we compare them through their variances (standard uncertainty squared), which is how they enter into the combined standard uncertainty, we get the following:

Source	**Standard Uncertainty**	**Std. Unc. Squared**	**Ratio to Largest**
Test Gage Temp. (steel)	690 nm	476100	1
Master Gage Temp. (steel)	690 nm	476100	1
Scale Temp. (glass)	400 nm	160000	0.34
CTE (scale)	20 nm	400	0.001
CTE (master gage)	35 nm	1225	0.003
CTE (test gage)	35 nm	1225	0.003
Length of Master Gage	250 nm	62500	0.131
Scale Specification	150 nm	22,500	0.047
	SUM	1,200050	

Since any source of uncertainty that is less than $^{1}/_{3}$ to $^{1}/_{4}$ of the largest component does not significantly contribute to the combined standard uncertainty, we see that there are three sources of uncertainty that dominate our measurement (highlighted). Since they all are determined by the temperature measurement, we see that we need a better thermometer. So, we buy a portable thermometer that we can put on or near the gages when they are being measured and a specification of 0.1 °C..

4. Example 2: 100 mm test ring gage calibrated using a 100 mm master ring on a long range ULMM. Lab has portable thermometer.

The typical uncertainty budget for this measurement equation looks like:

Master Gage	100 mm ring gage	uncertainty 500 nm (k=2)
ULMM spec	0.2 µm + 0.5 x 10^{-6} L	"accuracy specification"
Test Gage Material: steel	CTE = 12 x 10^{-6} /ºC	uncertainty 10%
Master Material: steel	CTE = 12 x 10^{-6} /ºC	uncertainty 10%
Scale Material: glass	CTE = 7 x 10^{-6} /ºC	uncertainty 10%

In addition:

- Test gage and Master gage temperatures are measured and corrected for using a digital thermometer; uncertainty specification is ± 0.1 ºC.
- Average temperature during measurements = 20.45 ºC.
- Scale temperature cannot be measured.
- Room temperature variation is ± 1 ºC.

Source	Uncertainty / Range	Dist.	Std. Unc. of Factor	Sensitivity Coeff	Standard Uncertainty	Std. Unc. (µm)
Test Gage Temp. (steel)	0.1 ºC	rect	0.058 ºC	12 x 10^{-6} L/ºC	0.7 x 10^{-6} L	0.069
Master Gage Temp. (steel)	0.1 ºC	rect	0.058 ºC	12 x 10^{-6} L/ºC	0.7 x 10^{-6} L	0.069
Scale Temp. (glass)	1.0 ºC	rect	0.58 ºC	7 x 10^{-6} L/ºC	4.0 x 10^{-6} L	0.400
CTE (scale)	0.7 x 10^{-6} /ºC	rect	0.40 x 10^{-6} /ºC	0.5 ºC	0.20 x 10^{-6} L	0.020
CTE (master gage)	1.2 x 10^{-6} /ºC	rect	0.7 x 10^{-6} /ºC	0.45 ºC	0.31 x 10^{-6} L	0.031
CTE (test gage)	1.2 x 10^{-6} /ºC	rect	0.7 x 10^{-6} /ºC	0.45 ºC	0.31 x 10^{-6} L	0.031
Length of Master Gage	0.50 um	norm	0.25 µm	1	0.250 µm	0.250
Scale Specification	0.25 µm	rect	0.15 µm	1	0.150 µm	0.150
				Combined Standard Uncertainty		0.51
				Expanded Uncertainty (k=2)		1.02

Let's examine the biggest components of this uncertainty. If we compare them by their variances (standard uncertainty squared), which is how they enter into the combined standard uncertainty, we get the following:

Source	Standard Uncertainty	Std. Unc. Squared	Ratio to Largest
Test Gage Temp. (steel)	69 nm	4,761	0.03
Master Gage Temp. (steel)	69 nm	4,761	0.03
Scale Temp. (glass)	400 nm	160,000	1.0
CTE (scale)	20 nm	400	0.003
CTE (master gage)	31 nm	961	0.006
CTE (test gage)	31 nm	961	0.006
Length of Master Gage	250 nm	62500	0.39
Scale Specification	150 nm	22,500	0.14
	SUM	256844	

We have now cut our uncertainty quite a bit, but we still have an expanded uncertainty that is not very good (1.02 µm). It is obvious where next to improve our process: the scale. One way is to make the scale out of a material that has a very low coefficient of thermal expansion, like fused silica, or more engineered materials like Zerodur or ULE. Fused silica has a thermal expansion coefficient of 0.5 x 10^{-6}/ºC, which is considerably less than glass. There are a number of engineered materials less than 0.1 x 10^{-6}/ºC. All of these will help. Another way is to put a thermometer on the scale, and make corrections.

Another way that should help, but is problematic at times, is to have the scale be steel and measure steel parts. If the scale and parts are the same temperature, and they usually are closer in temperature to each other than the room variation, you could get a lower differential thermal expansion correction and therefore lower the uncertainty some. Unfortunately many instruments have the scale in a closed housing and it is difficult to document the temperature difference.

5. Example 3: 100 mm test ring gage calibrated using a 100 mm master ring on a long range ULMM that has a low thermal coefficient material as the scale. Lab has portable thermometer.

The typical uncertainty budget for this measurement equation looks like:

Master Gage	100 mm ring gage	uncertainty 500 nm (k=2)
ULMM spec	0.2 µm + 0.5 x 10^{-6} L	"accuracy specification"
Test Gage Material: steel	CTE = 12 x 10^{-6} /ºC	uncertainty 10%
Master Gage Material: steel	CTE = 12 x 10^{-6} /ºC	uncertainty 10%
Scale Material: low CTE	CTE = 0.1 x 10^{-6} /ºC	uncertainty 10%

In addition:

- Test gage and Master gage temperatures are measured and corrected for using a digital thermometer; uncertainty specification is ± 0.1 ºC.
- Average temperature during measurements = 20.45 ºC.
- Scale temperature cannot be measured.
- Room temperature variation is ± 1 ºC

For our example, we will use a fictional engineered material that has a known thermal expansion of 0.1 x 10^{-6}/ºC ± 0.05 x 10^{-6}/ºC. Our table now looks like:

Source	Uncertainty / Range	Dist	Std. Unc. of Factor	Sensitivity Coeff.	Standard Uncertainty	Std. Unc. (nm)
Test Gage Temp. (steel)	0.1 ºC	rect	0.058 ºC	12 x 10^{-6} L/ºC	0.69 x 10^{-6} L	69
Master Gage Temp. (steel)	0.1 ºC	rect	0.058 ºC	12 x 10^{-6} L/ºC	0.69 x 10^{-6} L	69
Scale Temp. (glass)	1.0 ºC	rect	0.58ºC	0.1 x 10^{-6} L/ºC	0.057 x 10^{-6} L	6
CTE (scale)	0.01 x 10^{-6} /ºC	rect	0.006 x 10^{-6} /ºC	0.5 ºC	0.003 x 10^{-6} L	0.3
CTE (master gage)	1.2 x 10^{-6} /ºC	rect	0.7 x 10^{-6} /ºC	0.45 ºC	0.31 x 10^{-6} L	31
CTE (test gage)	1.2 x 10^{-6} /ºC	rect	0.7 x 10^{-6} /ºC	0.45 ºC	0.31 x 10^{-6} L	31
Length of Master Gage	500 nm	norm	250 nm	1	50 nm	250
Scale Specification	0.25 µm	rect	150 nm	1	150 nm	150
				Combined Standard Uncertainty		310
				Expanded Uncertainty (k=2)		620

We now have only one large component we can change, the uncertainty of the master ring. Suppose we send the ring gage to NIST for calibration. At NIST the ring is calibrated on our M48 coordinate measuring machine in a laboratory that is temperature controlled to 0.01 °C. The one directional repeatability of the M48 is less than 0.03 µm, and our long term reproducibility studies shows a calibration uncertainty for a 100 mm ring gage to be 0.12 µm.

With the development of long range instruments like the ULMM, the need for matching the master gage to the test gage is no longer necessary. This opens up the opportunity for labs to use the calibration services at NIST for labs that cannot afford the calibration of a whole gage block set. For nearly all size ring gages only a few masters are needed.

6. Example 4: 100 mm test ring gage calibrated using a 100 mm master ring on a long range ULMM which has a scale made of a low CTE material. Lab has portable thermometer. The master ring is calibrated at NIST.

The typical uncertainty budget for this measurement equation looks like:

Master Gage	100 mm ring gage	uncertainty 120 nm (k=2)
ULMM spec	0.2 µm + 0.5 x 10^{-6} L	“accuracy specification”
Test Gage Material: steel	CTE = 12 x 10^{-6} /ºC	uncertainty 10 %
Master Material: steel	CTE = 12 x 10^{-6} /ºC	uncertainty 1 %
Scale Material: ULE-like	CTE = 0.1 x 10^{-6} /ºC	uncertainty 10 %

In addition:

- Test gage and Master gage temperatures are measured and corrected for using a digital thermometer; uncertainty specification is ± 0.1 ºC.
- Average temperature during measurements = 20.45 ºC.
- Scale temperature cannot be measured.
- Room temperature variation is ± 0.1 ºC.

Source	Uncertainty / Range	Dist	Std. Unc. of Factor	Sensitivity Coeff.	Standard Uncertainty	Std. Unc. (nm)
Test Gage Temp. (steel)	0.1 ºC	rect	0.058 ºC	12 x 10^{-6} L/ºC	0.7 x 10^{-6} L	69
Master Gage Temp. (steel)	0.1 ºC	rect	0.058 ºC	12 x 10^{-6} L/ºC	0.7 x 10^{-6} L	69
Scale Temp. (glass)	0.1 ºC	rect	0.058 ºC	0.1 x 10^{-6} L/ºC	0.006 x 10^{-6} L	0.6

CTE (scale)	0.01 x 10^{-6} /ºC	rect	0.006 x 10^{-6} /ºC	0.5 ºC	0.003 x 10^{-6} L	0.3
CTE (master gage)	1.2 x 10^{-6} /ºC	rect	0.7 x 10^{-6} /ºC	0.45 ºC	0.31 x 10^{-6} L	31
CTE (test gage)	1.2 x 10^{-6} /ºC	rect	0.7 x 10^{-6} /ºC	0.45 ºC	0.31 x 10^{-6} L	31
Length of Master Gage	120 nm	norm	60 nm	1	60 nm	60
Scale Specification	0.25 µm	rect	150 nm	1	150 nm	150
				Combined Standard Uncertainty		193
				Expanded Uncertainty (k=2)		386

At this point, improvements become harder because there are a number of thermal components of about the same size, and the largest is from the ULMM. Fixing only one will not gain much.

Basically, for most calibration laboratories, this is the end of the road. If you look at industrial interlaboratory test data for ring or plug gage measurements, you see things like those in Fig. 1. [6]

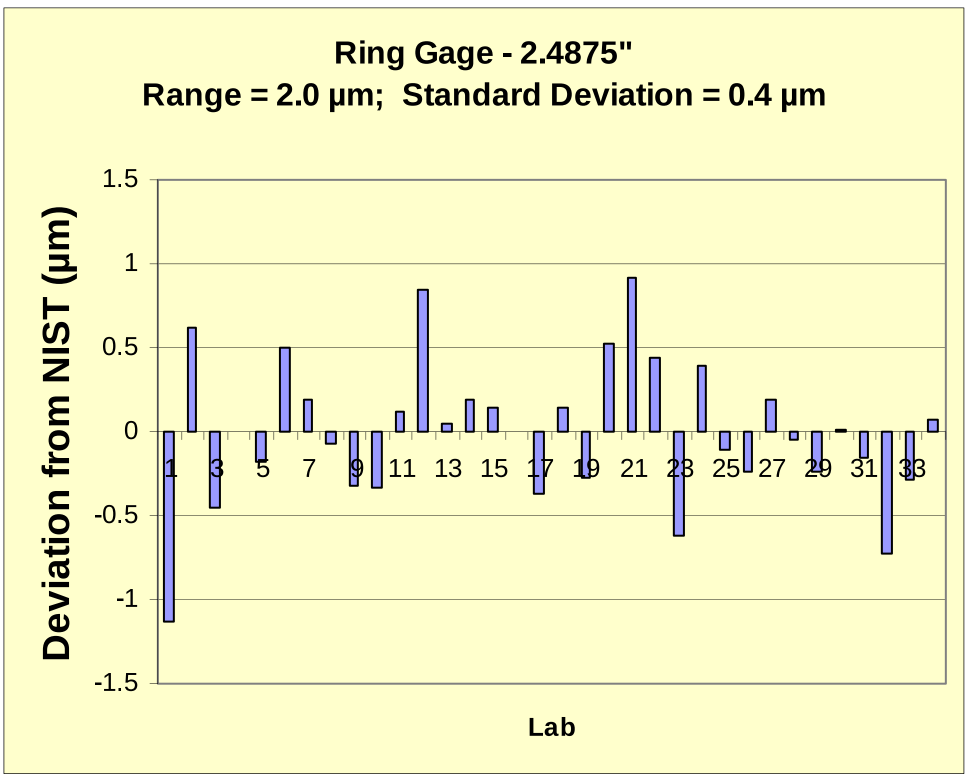

Figure 1: Deviation from the NIST calibration value for a 2.4875” ring gage as measured by 34 laboratories. The range of the data is 2.0 µm, with a standard deviation of 0.4 µm.

The results in Fig. 1 are typical of a round robin for randomly selected calibration labs. In this round robin, there were two rings and the deviations are the differences from the NIST calibration. The standard deviation between the laboratories was, in all cases, around 0.4 µm. If we take this as a rough estimate of the standard uncertainty of the group, we would get a k=2 expanded uncertainty of around 0.8 µm. According to our uncertainty budget in Example 3, this is about what we would expect for measurements made in an "average" environment (± 1 ºC) with decent thermometers (± 0.1 ºC) using a ULMM with a low expansion material scale.

7. Summary

Thermal expansion is a critical part of uncertainties for dimensional measurements. A fairly simple analysis of the uncertainty components can be used to decide how to rationally allocate resources to obtain adequate measurement performance.

8. Acknowledgment

I would like to thank Richard Pettit and the anonymous reader for pointing out a remarkable number of numerical errors in this paper. The paper is much stronger for their efforts.

9. References

[1] Strangely, few metrology books even mention thermal expansion. A survey of over 50 books on dimensional metrology or inspection revealed only two with mentions of thermal expansion, and those had only one paragraph each. The best source for information on thermal expansion is ASME/ANSI B89.6.2, *Temperature and Humidity Environment for Dimensional*. It contains details on all of the concepts used in this paper.

[2] International Organization for Standardization (ISO), *Guide to the Expression of Uncertainty in Measurement*, Geneva, Switzerland, 1993.

[3] American Society for Mechanical Engineering, ASME/ANSI B89.1.9, *Gage Blocks*, New York, NY, 2002.

[4] There are a large number of reference books that list the CTE of various materials. Unfortunately the uncertainty is seldom reported, and the CTEs reported in most sources are averages over large ranges of temperature, which increases the uncertainty at 20 °C. The 10% used in this paper is the consensus value used by experts in the field.

[5] How a 1D measuring machine came to be known as a "universal" measuring machine is not known, but probably results from the fact that it can be used to measure both internal (ring) and external (plug) dimensions. A machine that measures in three dimensions is occasionally called a "universal measuring machine," but the most common term is "coordinate measuring machine."

[6] Private correspondence.