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Measurement Assurance Program— A Case Study: Length Measurements. Part 1. Long Gage Blocks (5 in to 20 in)

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A Case Study: Length Measurements.
Part 1. Long Gage Blocks (5 in to 20 in)

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Contents

The Measurement Assurance Program—A Case Study: Length Measurements

Part I—Long Gage Blocks (5 in to 20 in)

P. E. Pontius

The differences between the methods of traditional metrology and the measurement assurance programs are briefly discussed. The historical data relative to long gage blocks (5 in to 20 in) are analysed to provide a basis for comparison with results from new measurement processes formulated in accordance with the philosophies of the measurement assurance programs. The results from the new processes are in agreement with the work of the past. The current length values assigned and associated uncertainties are shown for selected long gage blocks used in the dissemination of length by the National Bureau of Standards. These long gage blocks are a part of a growing collection of similar well characterized artifact standards for use in comparative measurement processes. The methods and techniques used in developing the new measurement process are discussed in some detail. It is the author's intent that, in addition to the technical content, this paper be largely tutorial in the area of measurement process analysis. This paper is, in essence, a report on the extension of the techniques first suggested in NBS Monograph 103 "Realistic Uncertainties and the Mass Measurement Process" to the area of length measurement.

Key words: Measurement algorithm; measurement assurance; measurement process; measurement unit; process variability; uncertainty.

1. Introduction

The National Bureau of Standards has been engaged for some time in the development of length measurement processes which are in accordance with the philosophies of the Measurement Assurance Program (MAP). One of the ultimate goals in this work is the optimization of the uncertainty [1] of the values assigned to artifact length standards, such as gage block. Work to this end requires not only a reevaluation of the manner in which values are assigned, but also a complete characterization of the measurement processes of both the National Bureau of Standards and the users of the calibration service. Eventually all of the National Bureau of Standards length measuring processes will be modified. This paper covers the progress to date on the long gage blocks (from 5 to 20 in in length).

Currently, the basis for the values reported by the National Bureau of Standards are the values assigned to a group of gage blocks which are normally called "Starting Standards." These values have been assigned by interferometric methods using a stabilized laser as a light source. The wavelength of the laser light has been established, and is monitored periodically with indirect reference to the present defining Krypton radiation.[2] Values assigned to other blocks are determined from comparative measurement data. At the present time, comparators with contacting transducers are being used, however, any well characterized comparative process would be suitable. All new procedures include features which permit the establishment of meaningful process performance parameters, as well as means to monitor the process performance over time. The purpose of this paper is to verify the closure between the "old" process and the "new" process, and to describe the present "points of departure" upon which some of the current assigned length values are based.

This paper is organized as follows: Section 2 presents some of the philosophical differences between traditional metrology and the measurement assurance programs. Section 3 examines in detail a typical list of "error budget" items in the light of the measurement assurance program philosophy. Section 4 is a review of the calibration history of selected gage blocks. The historical data are analysed to establish a predicted value at a given time and an estimated uncertainty of that value. The predicted value will be used to verify continuity with the "new" process results. Section 5 discusses

[1] The term uncertainty is used to designate a quantitative statement of the bounds for error assoicated with a particular measurement result. An optimized uncertainty results from a selection of measurement methods and processes which, in combination, conserve measurement effort yet provide demonstrable evidence that the uncertainty is realistic regardless of the number of transfers between the user and the defined standard. A realistic uncertainty is, in turn, the basis for judging the adequacy of a given measurement result relative to the manner in which the result is to be used.

[2] Direct calibration of the laser wavelength against 86Kr is possible, but is relatively tedious and expensive. The procedure used is a heterodyne comparison of the stabilized He-Ne laser with an iodine stabilized laser. This procedure is both rapid and precise, with measurement errors at or below 1 part in 10^9 easily achieved [1].

the new procedures, the interferometric and comparative process, and also some of the supplementary studies relating to systematic errors. Section 6 demonstrates that there is continuity and discusses the development of the "new" measurement process. Section 7 summarizes the performance to date of this "new" process. Section 8 is a closing summary.

2. A Comparison—Traditional Metrology versus Measurement Assurance Programs

Traditional metrology, almost exclusively concerned with the propagation of measurement units, regarded measurement as a realization of the highly idealized process by which the quantity being measured was defined. Each measurement was in essence a "work of art," the result being accepted mainly on the basis of the method used and the reputation of the person making the measurement. The traditionalist knew that placing severe restrictions on the characteristics of objects being measured, and on the measurement environment, would reduce errors from such sources. While his stature among his peers was directly related to the smallness of the uncertainty associated with his work, he did not always have the means to establish realistic uncertainty statements. When his measurements related to practical measurement processes, in many cases, his results were considered correct by "definition." His uncertainty statements were largely a matter of judgment, sometimes with a disclaimer stating that the result was applicable only in his laboratory at the time of the measurement. Nonetheless, while he had difficulty in relating to practical measurements, his diligence and attention to detail provide a basis for a different approach.

The dividing line between the era of traditional metrology and the era of measurement assurance programs is clearly associated with the development and increasing accessibility of large computers. Even with simple comparison procedures, the traditionalist had to make long, detailed hand computations. A large percentage of his time was spent in concentrating on mathematical procedures, checking and double checking hand computations. In order to simplify the computations, tables were constructed for a variety of things such as air density, and "fringe fractions" per microinch departure from nominal value for a variety of spectral sources under standard conditions. As a consequence the measurement processes became married to the tables. Now, through the computer, one has in effect tables for all possible combinations of variables. In addition, matrix manipulation, statistical analysis, control charts, correlation studies, and the like are immediately available. Both the philosophy and scope of the measurement assurance programs are a direct result of being able to store and recall large amounts of data, and to analyze and format the results in many different ways.

One can think of all of the measurement processes in our technical society as parts of measurement systems—length measurement systems, mass measurement systems,[3] and so on. Within these systems, measurements are made to accomplish a wide variety of functions. The measurement processes are merely tools, subject to an equally wide variety of value judgments. It is the quality of the measurements made by these processes that are of primary concern. One would like to have assurance that each individual measurement within the system is correct enough for its intended use. When the measurement result is necessary to the successful completion of the task at hand, then if length or mass (or whatever) measurements are to have real meaning, one must be able to make the required measurements on the object of interest in an environment appropriate to the particular task.

The measurement process is a production process, the product being numbers which represent certain characteristics of the item or phenomenon under study. The uncertainty at a given location is related to the output of the whole process—the instrument, the operator, the procedure, etc. Once one accepts the concept of measurement as a production process rather than a "work of art," one can introduce redundancy into the procedures which can be used to ascertain parameters which are descriptive of the process performance characteristics and to monitor the process performance.

For example, comparing one gage block with two "master" blocks, and averaging the results, will establish a number for the length of the block. In the past the redundancy of this method was used to check for "bad" measurements and almost never to establish properties of the process. Such a procedure could also give information about the process, namely the observed difference between the two "masters." Comparing two gage blocks with two "masters" according to a measurement design provides, with little additional effort, not only a value for each of the two blocks, but also a "check" on the constancy of the "masters," an estimate of the short term process precision, and in time, an estimate of the long term process variability. The first procedure in which a single gage block is compared to two "masters" requires only simple arithmetic operations. The second procedure requires sophisticated data processing which has only been available within the last ten years.

[3] The initial work leading to the development of measurement assurance programs in calibration was in the area of mass measurement. This work is described in part in references [2, 3, and 4].

The complexity of a measurement assurance program depends upon the purpose a particular measurement is to serve. The program analysis relies on data actually generated by each measurement process; therefore, the process performance parameters are valid descriptors of the expected process performance. Procedural steps incorporated in routine measurements provide data which, in turn, can be checked against parameter estimates to give a continuing assurance that the process performance is as expected. For processes supporting modest requirements, these procedural steps can be very simple. As the requirements become more stringent, obviously the monitoring procedures become more complex.

Assigning a value which is to be used as the length of a gage block and determining the uncertainty of that value is not a simple task. The properties of the block, as a function of time and temperature, are determined using a process whose behavior must be monitored as a function of procedural, instrumental, or environmental changes. In addition, there are parameters related to the optical or mechanical process which need to be measured to provide bounds for the departures from the assumed physical model. Also, one needs to have valid estimates for the various components of variability from which one can make the proper combination for bounds to random error. To achieve the objective, one needs a sequence of measurements over a sufficiently long time span to allow the influence of various perturbations to exert their full influence on the process. This leads to valid estimates of the effects of random error. By properly combining these with systematic study of the effect on the output of controlled changes in certain factors and correlation studies of the effects involving those factors not subject to control, one arrives at a realistic uncertainty statement.

Having completely characterized one process, in terms of process definition and appropriate performance parameters, the characterization of other similar processes consists mostly of determining the numerical values for the various performance parameters. These parameters determine realistic uncertainty statements which permit meaningful comparison of results from different locations.

The measurement assurance approach enables one to clearly establish the limitations of a particular method of measurement. In cases where the required uncertainty of measurement for a particular task is unusually small, the program will provide some guidance as to possible actions which may produce satisfactory results. The program provides a means to monitor the performance of various measurement processes throughout the system. One result from the program is a clear definition of the level at which various factors significantly affect the uncertainty. This information, together with a clear definition of process performance requirements, can be used to grossly simplify many existing measurement processes and procedures.

3. Variability—Two Approaches

As a point of departure, all length measurement processes are, directly or indirectly, comparative operations. Even the most simple concept of such a measurement contains certain implicit assumptions:

(a) a constancy in the basis for the ordering or comparing;

(b) a stability in the equipment, procedures, operator and the like which are used to make the measurement; and

(c) a stability in the object, effect or property being observed.

Quantitative ordering implies an invariant basis for the ordering, thus a long term constancy in a standard unit and a stability in the realization of a standard unit, is necessary. In a similar manner, the property to be measured must also be stable. If a measurement process detects a difference between two things, it is expected that repeated measures of that difference should agree reasonably well. In the absence of severe external influence, one does not expect things to change rapidly.

There is a difference between stability and constancy in context with the above. Repeated measurements over time can exhibit a random like variability about a constant value, or about a time dependent value. In either case, if the results are not erratic (with no unexpected changes), the process is considered to be stable. The objects being compared may have constant values, or may be changing at a uniform rate, or may be changing at different rates. For continuity, time dependent terms must be included in quantitative descriptors for both objects being compared. Stable changes with time can be extrapolated in the same manner that one "extrapolates" a constant value over time. The extrapolations can be verified whenever desired by making additional measurements. Constancy, then, merely means that the coefficients of time dependent terms are essentially zero. Gage blocks which are changing at a constant rate are considered to be stable. This is not to say that features such as constancy, and perhaps geometry, are not desirable for certain usage, but only that such features are not necessary restrictions on the ability to make good and useful measurments.

Two quantitative descriptors are used to describe the process variability, and ultimately, to establish the bounds for the limit of error. A given measurement process is continually affected by perturbations from a variety of sources. The random like variability of the collection of repeated measurements is a result of these perturbations. One descriptor, designated random error, includes effects from both cyclic perturbations such as might be associated with the environment and variability associated with operating procedures. The random variability implies a probability distribution for which one can set limits such that the range of variability in the collection is not likely to exceed certain bounds. The second descriptor, designated systematic error, S.E., includes the use of constants which are in error as well as discrepancies from certain operational techniques. The S.E., expressed as a single number, is an estimate of the offset of the measurement result from some defined process average. These two descriptors, called the process performance parameters, are factors in assessing the worth of a result relative to a particular requirement.

The random error estimate reflects the effects of cyclic perturbations which are constantly changing whether the process is being used or not.[4] These effects can be grouped into two categories: short term effects which vary through one or more cycles in the course of a single measurement or measurements made over a short time interval, and long term effects in which the period of the effect is at least as long as the time required for a given sequence of measurements.

A second category of short term effects are those which are instantaneous, or step-like, in nature. In many cases, "shocks" on the instrument, or variations in the manner in which various objects are introduced to the instrument, cause changes in the instrument configuration which affect the instrument indication. The effects appear as minute, and sometimes not so minute, instrument reading scale shifts. The thickness of the film between two gage blocks which have been "wrung" together is an example of a step-like source of variability. While for each "wring" there is a finite film thickness, for repeated "wrings," the film thickness is never quite the same.

In terms of measurement process performance, the within-group variability expressed as a standard deviation, σ_W, reflects the combined short term effects. In many cases, σ_W represents an optimum process performance. The within-group variability of the measurement process is the most familiar process parameter as it is easily demonstrated in a repeated sequence of measurements of the same thing in a very short time interval. Practically all important measurements are repeated several times. The magnitude of the within-group variability is generally established by the degree to which certain types of perturbations are controlled and by factors such as the operator skills, quality of the instrument, and attention to detail procedure. In most cases one cannot identify sources of perturbations which contribute to within-group variability. Process improvement in terms of reducing σ_W is obtained perhaps more frequently by trial and error than by design. The adequacy of a given process relative to a particular requirement is often judged on the basis of the within-group variability. Such a judgment, however, may be erroneous.

The total variability is the variability of a long sequence of data which reflects the effects of all possible perturbations. Repeating a given measurement over a time interval sufficiently long to reflect the influence of all possible perturbations establishes a total process standard deviation, σ_T, which reflects both the short term and the long term random variability.[5]

With a sufficiently long sequence of data, one should be able to identify the sources of the largest perturbation through supplemental measurements and correlation studies. Having identified the source of the largest perturbation, the magnitude of its effect on the measurement can be minimized, with a consequent reduction in the magnitude of σ_T. Frequently one is tempted to idealize the process in order to reduce the total variability, that is, to establish a carefully controlled environment and use only selected artifacts. Such actions are self-defeating in terms of understanding the measurement process. A more appropriate action, provided one has sufficient motivation and resources, is to modify the process to account for the variability associated with all the perturbations that can be identified.

There are several different classes of Systematic Errors. Perhaps the most familiar class of S.E. is associated with instrument reading scale offset. Such S.E.'s are not present in comparative measurements provided that the instrument indication can be related to the measurement unit, and provided that the instrument response is reasonably linear over the range of difference which must be measured. A second class of S.E.'s is associated with supplemental data such as barometric pressure, temperature and relative humidity measurements which are in turn combined to determine air density, index of refraction and the like. Each of the supplemental measurements is, in essence, a separate distinct measurement process with both random variability and systematic effects. The

[4] Reference [5] and figure 24 in section 7.1 show that the collection of values obtained by sampling at random times the value of the sum of as few as four sinusoidal functions, each of equal amplitude, but of periods differing by a factor of 10, has the appearance of a normal distribution for even moderate sized sequences of observations.

[5] The total process variability, σ_T, can be thought of as the sum of the variabilities of all of the perturbations that affect the process, that is, $\sigma_T^2 = \sigma_1^2 + \sigma_2^2 + \ldots \sigma_n^2$. For one class of perturbation with variabilities σ_1 to σ_m, which are those with very short periods and with nearly equal amplitudes, it may not be possible to identify the individual perturbations. The variability from these perturbations combine to form a threshold variability σ_w. Other perturbations, with variabilities σ_{m+1} to σ_n, may be identifiable if the magnitudes are sufficiently large. These effects combine to form a between time component of variability σ_β. The total variability is then $\sigma_T^2 = \sigma_w^2 + \sigma_\beta^2$.

random variability of the supplemental measurements is, of course, reflected in the total process variability. The S.E.'s associated with supplemental data must be carefully considered.

One action, which is rarely practical, would be to "randomize" the S.E. by using different instruments, operators, environmental or other factors. Thus, the variation from these sources becomes part of the random error. A more practical procedure is to evaluate the S.E. associated with an instrument (or other factor) by direct experiment. When the change in response, such as, for example, that introduced by a temperature error of 0.1 degree, is a small fraction of the standard deviation of the process, a rather large number of measurements is required to establish the effect with a reasonable degree of assurance. Bearing in mind that an average of n measurements has a standard deviation of $1/\sqrt{n}$ times that of the original measurements, in order to determine an effect of size one standard deviation with an uncertainty (3 standard deviations) of half of its size one would need about 36 measurements. (If one relaxes the uncertainty requirement for the average to a value equal to the standard deviation of the process, then 9 measurements would be required.)

With evidence that the individual supplementary measurements are satisfactory, the next concern is the manner in which supplementary data are combined and used to adjust the observed data. For example, having adjusted the data for thermal expansion, one would not expect a collection of values over time to correlate with the temperature measurements for each individual value in the collection. A collection of values from repeated measurements should be tested for correlation with each of the supplementary measurements, and their various combinations, as appropriate. If correlation is indicated, either the supplementary measurement is not being made at the appropriate location, or the manner in which the supplementary measurements are combined does not describe the effect that is actually occurring. Corrective action is necessary. Low correlation does not necessarily indicate that there are no S.E.'s present, but only that for the supplementary measurements which have been made, the magnitude of the combined S.E.'s is not large relative to the total standard deviation of the process.

There may be long term systematic error effects from sources not associated with the current supplemental measurements. It is relatively easy to demonstrate the presence or absence of such effects, but it may be difficult to reduce their magnitudes. If one has available a collection of values over a long time span, one can compare the standard deviation as computed for small numbers of sequential values over short time spans with the standard deviation of the total collection.[6] While reasonable agreement is expected, frequently such is not the case. If the magnitude of the effect is sufficiently large, the collection of values may indicate grouping, with the group means appearing as random variability about the process average. If the distribution of the collection of values appears to be bi-modal, one should look for a large long term cyclic effect. Until the source of such variability is identified, and appropriate action taken to modify the process, the total standard deviation must be used as the descriptor of the random variability of the process.

The purpose for measuring gage blocks is to assign numbers representing the lengths of the blocks in such a way that the numbers will be useful to others. The reason for characterizing the measurement process is to assign meaningful error bounds, or uncertainties, to the numbers representing the lengths. The magnitude of the uncertainty is established by the error bounds of the local measurement process and the error of the accessible unit. In most mass and length measurements, access to the unit is through an artifact which has been assigned a length, or mass, value by another measurement process. In the case of mass, for example, the international prototype kilogram is defined to have zero unit error. With a proces operating in a state of control, that is, with no known systematic effects unaccounted for, and with the international prototype kilogram to introduce the unit, the uncertainty is only a function of the process standard deviation, either σ_W or σ_T.

One may report a single measurement, or the average of n measurements. Few, however, can afford the time and effort to make a very large number of measurements. As a consequence, the "reported" result is always offset from the process average by some amount. This offset is called a systematic error and can be either plus or minus. When the object as measured above and its assigned value are used to provide access to the unit in another process, this systematic error, which is associated with the unit, in combination with the random variability of the second process, is the uncertainty of the result from the second process. For all well characterized measurement processes, the S.E. associated with the accessible unit is the only S.E. component in the uncertainty, all other identifiable S.E.'s having been accounted for in the process.

Fortunately, most measurement processes for a given parameter are similar so that the complete

[6] The use of comparison designs, described later in this paper and discussed in detail in reference [6], facilitates this type of analysis. The within group variability, σ_W, is computed for the prescribed sequence of measurements. Each measurement sequence includes in effect a "check standard" which is measured over and over again with similar measurement. The total standard deviation is computed for the collection of values for the "check standard." The inequality $\sigma_T > K\sigma_w$ is taken as evidence of the existence of a long term systematic effect, perhaps as yet unidentified. The term K in the above relation accounts for the fact that the "reported" value of the "check standard" from the observations required by the design is not a "single value" but, in effect, is, the average of "n" measurements in the design sequence while σ_W is the standard deviation of a "single measurement."

characterization and documentation of a typical process over the range of objects and environments in which the measurements are usually made substantially shortens the time required for characterizing other processes. As a practical limit, few can afford the time and effort to identify perturbations related to either the between-group variability of S.E. components (as previously discussed) with effects of magnitude less than one standard deviation of the within-group variability. In the end, the uncertainty associated with a sequence of operations defined to be a measurement is determined in part by the larger of σ_W and σ_T and by the S.E. components associated with the unit. The uncertainty statement must also include the S.E.'s which are not accounted for in the measurement process for reasons of convenience.[7]

One traditional method for determining the limit of error, or uncertainty, of a measurement result is the use of an error budget. In this method, one compiles a listing of all known sources of error which might affect the measurement result. Table 1, [7],[7a] shows a rather complete list of the usual error budget items associated with measurement processes used to assign length values to gage blocks. In the traditional method, one makes a theoretical analysis of the algorithm and "engineering adjustments" to provide estimates of the magnitude of the expected variability term by term. Such estimates would then be combined in some manner to obtain an estimate of the total expected error bounds. While the error budget analysis may be helpful in some kinds of measurement, it is not unusual to find the results of measurements of the same thing which disagree in excess of the error bounds established in this manner. In a repetitive measurement process, such as the calibration of gage blocks, one can verify experimentally the magnitude of the significant effects contributing to the process variability.

The items normally considered in an error budget can be further developed into categories according to the way in which they are most likely to affect the uncertainty. One category would contain items which relate to S.E.'s; another category would relate to σ_W; and the third category would relate to σ_B or σ_T. To illustrate the nature of these categories, table 1 also shows a tentative disposition in terms of a measurement assurance program selected to: (1) disseminate a physical embodiment of a length unit; (2) characterize a measurement process in such a manner that realistic uncertainties can be established for the assigned values; and (3) provide a basis for sorting with respect to other properties desired for a particular usage (i.e., deviation from desired nominal value).

[7] In many cases acceptable limits relative to a particular usage are large with respect to measurement process capabilities. In the interest of conserving measurement effort, detailed corrections for S.E.'s are frequently ignored. When such is the case, the effect of the ignored S.E.'s must be included in the uncertainty statement.

[7a] Figures in brackets indicate references on page 54.

The items included in table 1 are divided roughly equally between being contributors to the within-group variability, and the between-group variability. All but two can be monitored or evaluated by a judicious choice of a comparison design, to be used over time so that all of the perturbations can exert their full effect on the measurement process. The two exceptions are the uncertainties associated with the assigned starting values, or restraint values, and the conversion of the present instrument indications to length units.

4. Summarizing History

4.1. Predicted Values

To assure continuity in the transition to a measurement process formulated on the basis of a measurement assurance program, some tie between the old and the new process must be established. If, for a given block, a predicted value based on historical data can be established, a reasonable estimate of the uncertainty of this value can provide a basis for comparison with a current value produced by a new process. The difference between the old and the new values relative to the uncertainty of each would clearly verify the continuity, or discontinuity, of the measurement system. To start, an analysis of historical data is necessary to establish an estimate of the predicted length for each block (as defined in appendix 1), at a specific temperature and at a specific time, together with an estimate of the uncertainty of that value. Such a task has been completed for two groups of long gage blocks (nominal lengths ranging from 5 in to 20 in).

The first set to be discussed includes the following blocks, the number following "NBS" being the serial number, and the dash number being the nominal length in inches and (.) being the short designator (read as "one dot"):

NBS – M136–5 (.)	NBS – M109A–10(.)
– M115A–6(.)	– M135A–12(.)
– W202A–7(.)	– M109A–16(.)
– M103A–8(.)	– A157–20 (.)

A cursory review of the calibration history indicates a reasonably stable (not erratic) condition since approximately 1956. Measurement data, if in existence prior to this date, were not considered in this analysis. Where necessary, the historical data have been adjusted to reflect redefinitions of the inch and of the practical temperature scale [8, 9]. For comparison, over this time period, the announced uncertainty associated with gage block calibration was $\pm 1\mu$ in per inch of length. It was privately felt that a more realistic estimate might be $\pm 5\mu$ in for 5 in through 10 in, $\pm 6\mu$ in for 12 in, $\pm 8\mu$ in for 16 in and $\pm 10\mu$ in for 20 in. With few exceptions, for the individual blocks, the deviations from the fitted line shown are well

TABLE 1
Error Budget

	Item	Disposition
Spectral Sources Krypton 86	"Red-orange line exact by definition limited by practical considerations to about .01 ppm in vacuum."	Used only in scanning interferometer under conditions which realize the defined length as closely as possible. Defined out.
	"Spectral lines necessary for exact fraction interferometry."	Exact fraction interferometry used only to determine integral fringe order. Discarded.
	"Lamp construction features" "Lamp operating conditions" "Change with age, stability"	Lamp performance monitored relative to stable lasers. A problem only to those who assign vacuum wavelengths to other spectral sources. Discarded.
Stabilized Laser (Lamb-dip)	Uncertainty associated with the assigned vacuum wavelength.	Vacuum wavelength assigned on the basis of comparison on scanning interferometer with red-orange line of Kr 86. Periodic checks verify stability. Uncertainty of no practical concern with the U.S. intercomparison of stable lasers on international basis under way, with undoubted outcome of replacing the Kr 86 definition. S.E. contribution negligible.
Working Sources	"Accepted vacuum wavelengths determined by limited experimental measurements and reproducibility. Hg 198 - .05 ppm; Cd 114 - .07 ppm. Lamp construction features. Lamp operating conditions."	Used only in exact fraction interferometry to determine, or verify the integral fring order number when necessary. Discarded.
Index of Refraction Standard Conditions	"Conversion vacuum wavelength to standard conditions using Edlen, Barrell and Sears relation. Alternate method using refractometer having uncertainties dependent on use and design."	Use latest assessment of Edlen formulas, converting directly from vacuum to actual conditions. Error in functional form systematic to whole system. Verify by closure.
Standard to Actual Conditions	"Conversion to actual for small range of variable, if spectral dispersion is used rather than approximations, equations can be considered to introduce negligible errors. Errors from measuring environmental conditions are:"	This group of items is considered to be a source for between-group variability. It is assumed that, by using the calibration data, the observed data would produce results that tend to a limiting mean which would not differ significantly from the real value. That is, the desired parameter such as air temperature is nearly as often
Index of Refraction Standard to Actual Conditions	a. Barometric pressure error (mmHg)x0.36ppm; typical top quality mercury manometer .05 cal. uncertainty + .05mm sd reading; (.05+3x.05).36=.07ppm. b. Air temperature error, (deg C)x.93ppm; 05° typical thermometer cal. uncertainty + .015sd reading; (.05+3x.015)=.09ppm.	indicated high as well as low, and the process can detect very small changes. Correlation studies between the measured temperature and the final results will indicate the presence of significant variability due to air temperature measurement problems locally. Closure tests may indicate systematic differences between different measurement processes. The same applies to barometric pressure and humidity. Between component.
	c. Humidity (vapor pressure) error (mmHg)x.05 ppm; typical cal. uncertainty 1mm, reading error negligible (5% rh); 1x.05-.05ppm. d. CO_2 content assumed standard usually. e. Other impurities in air which affect index of refraction.	Insofar as CO_2 content and other impurities, local process variability, either long or short term, relates to the difference between the local environment and the assumed standard air. With sufficiently precise processes, closure test might show up differences between processes, if it were possible to determine the composition of the local environment in an easy way. The nature of these changes is only now beginning to be studied as a part of pollution studies. Verify by closure.
Interferometer Instrument	"Slit width and obliquity."	Use best recommended formulas, if significant relative to precision of process. Could be checked with closure studies.
Environment and Setup	"Alignment, angle of incidence; dimensional stability of instrument; vibration, illumination."	First consideration is magnitude of effect relative to a precision of process. If effect significant, must adjust carefully. Other factors affect the within-group variability.
Interpreting Interference Pattern	"Planeness of wave front; symmetry of interference pattern; (coatings, noncompensation, obliquity); linearity of viewing optics; contrast and light level of interference pattern; method of estimation of fringe fraction."	Certain items are obviously a part of the instrument design and the presence, or absence, of significant effects can only be determined by closure. The use of the laser light source eliminates many of the problems of the past. Fringe interpretation by means of photo-interpretation allows a more versatile approach to the problem. The present procedure uses multiple points to extrapolate to the gaging point. The use of scanning equipment may permit determining departures from a defined point at other selected points. Fortunately, the interpretive process is relative, thus as long as film changes do not destroy the relative position of the image, there is no problem. Within-group component.

TABLE 1 (continued)

	Item	Disposition
Interference Length of Block		
Phase	"Basic material properties, apparently due to finish."	Select procedures to minimize. If departure from defined process introduces significant changes, be prepared to determine and adopt appropriate correction factors. Verify by closure.
Wringing Film		Defined out. Process variability affected in part by variability of wringing film. Studies under way to try to characterize wringing film, the mechanism, and to develop techniques to minimize variability.
Coefficient of Thermal Expansion		Precise comparative processes will allow experimental determination of coefficient of expansion for each block with sufficient precision to be useful over a wide range of temperature. While it is conceded that a reasonably constant temperature is desirable during the course of a series of intercomparisons, only convention says that the temperature must be 20 °C. Between component.
Temperature	"Realization of IPTS; temperature system calibration; physical thermal contact thermometer to block."	Variability block to environment affects between-time variability. Temperature differences between blocks during comparison affects within-group variability.
Compression, Bending	"Atmospheric pressure; mechanically applied forces; gravity forces; magnetic forces; material constants such as Young's Modulus, Poisson Ratio."	Minimized by definition of attitude of block at time of measurement. Magnitude predictable and of concern only when significant relative to process precision. These items perhaps more troublesome in the area of dimensional technology. Defined out.
Geometry	"Parallelism effect on reading and definition of length; flatness; geometry of interface with wringing."	Only real constraint on parallelism is that associated with determining fringe fraction. Photointrepretation permits a much wider latitude for variation than interpolation by direct observation. Flatness, and interface, affect the attitude of the block at the time of measurement. Failure to reproduce in attitude is a between-time component of variability.
Mechanical Comparison		
Stylus	"Deformation, material constants and variation in surface finish."	For blocks of similar materials, contributes only to the within-variability. For blocks of different materials, contributes to total variability. Data adjustment may be necessary.
Temperature	"Temperature coefficient, temperature difference."	Contributes only to the within-group variability. Judicious choice of comparison design will minimize effect. Can measure temperature differences and correct if necessary.
Forces	"Bending and compressive forces, variation in material constants, compression clamps."	In a comparative operation, contributes to within-group variability only.
Stability	"Stability of apparent length with time of comparison of both block and instrument."	Minimized by judicious choice of design. Contributes to within-group variability.
Environment	"Adverse environment conditions; rapid thermal lag problems; vibration electrical interference, etc.	Either defined out, or brought under control in initial assessment of process performance.
Design	"Anvil-stylus design relations to length definition, effect of flatness of block and anvil."	Minimized by comparative operation. Contributes to within-group variability.
Geometry	"Effect of parallelism errors of block on measurement regarding gage point definition and interaction to squareness of sides to ends."	By specifying the measurement and the attitude of the block at the time of measurement, this is defined out. Further tests may be necessary to establish suitability of block for uses other than transfer of length unit.
Alignment	"Effects of alignment of instrument measuring tips; block seating, burrs, parallelism errors."	Alignment effects minimized by comparative operation. Block seating variability contributes to within-group variability. De-burring, etc. a part of a definite "premeasurement" procedure.
Indication	"Magnification of comparator indication." Establishing conversion indication to length units.	Magnification must be such that some variability is indicated in repeated measurements of the most stable object. Conversion of scale units to length units cannot readily be made a part of the measurement at this time. Methods are under study which will permit verification in the course of the measurement. For the present, this must be accepted as a systematic error. The magnitude of the error may be insignificant over the range of small differences normally encountered if the calibration is done carefully.

within these limits indicating that perhaps more care than normal was exercised in assigning values to these blocks.

Figure 1 shows assigned values for the above blocks over the time interval 1956 and 1971. The values shown are expressed as corrections or departures from nominal length where the assigned length, L, equals the nominal length, N, plus the correction Y_N. The assigned values were used as "exact," as indicated by the straight dashed lines, until such time that a later value was determined. The points are spaced along the X axis according to the date of the particular document from which they were obtained. Figure 2

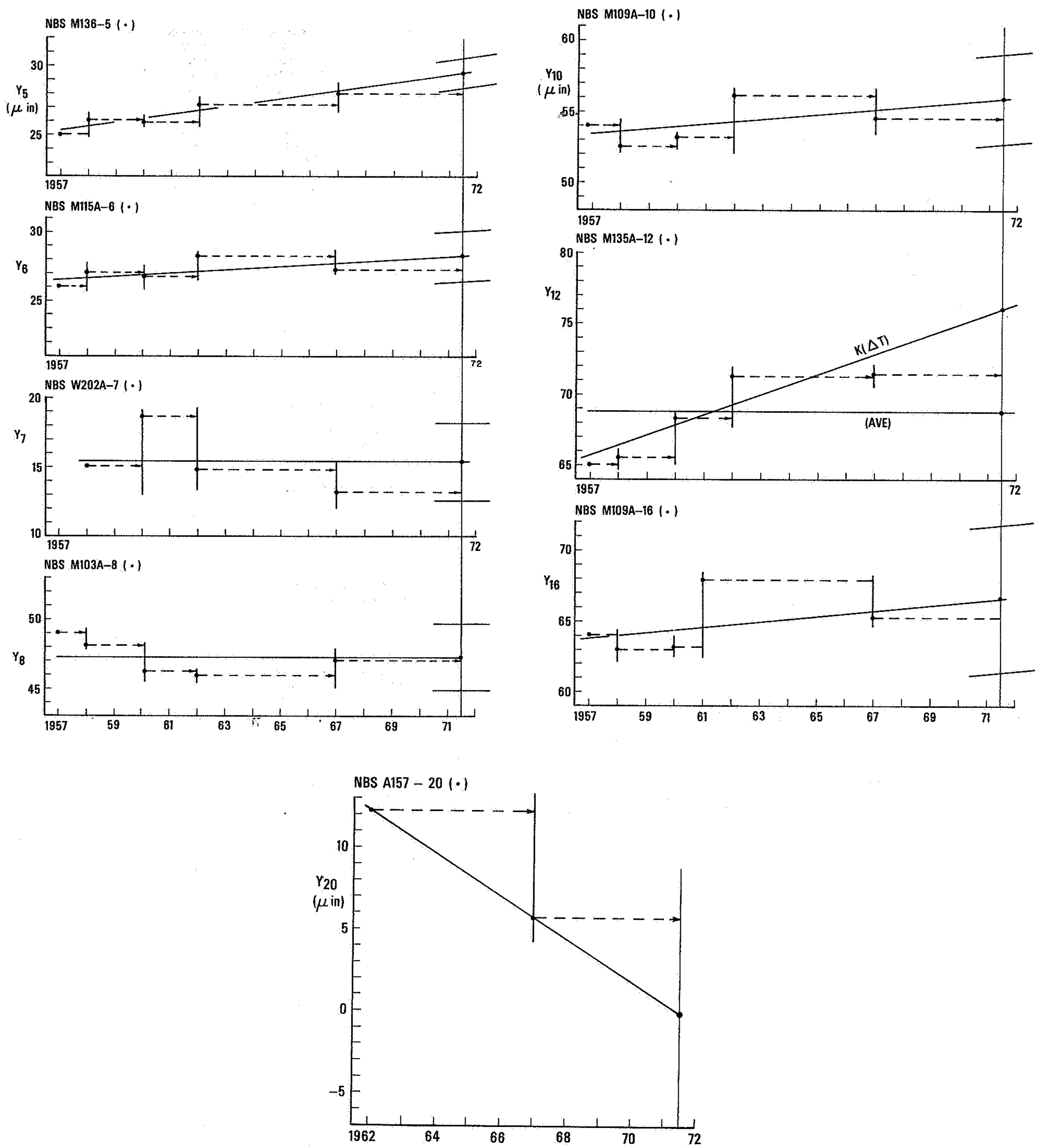


FIGURE 1. *Historical assigned values, NBS(.)*

shows the measurement history for a similar set, the USN blocks, over the same time interval [10]. Again, the deviations from the fitted line are well within the previously stated limits.

The values shown were computed from data produced by an interferometric measurement process which utilized light sources of different wavelengths. (For the purpose of this report, such a procedure is called multiple wavelength interferometry.) While it is known that varying amounts of measurement effort are associated with the values shown in these figures, it is accepted that each value is the result of work which was done by careful, dedicated metrologists and, as a consequence, there is no reason to believe that any one is more reliable than the others. As a rule, each block was monitored by comparison with other known blocks, therefore no particular time increment was used to determine when a "recalibration effort" was required.

The general patterns of the historical values for both the NBS (.) blocks in figure 1 and for the USN blocks in figure 2 indicate a constant change in length over the time period covered. In order to establish an appropriate tentative "predicted value," a line of the form:

$$(Y_N, t) = (Y_N, t_o) + K_1 (t - t_o)$$

was fitted to the data. In this relation the correction to nominal length at any time t, (Y_N, t), is a function of the correction at an arbitrary time, t_o, (Y_N, t_o), the rate of change in microinches per year, K_1, and the time interval in years $(t - t_o)$.

In the case of the NBS 7(.) and 8(.), as shown in figure 1, and the USN-7, shown in figure 2, it was decided that $K_1 = 0$, thus the predicted value at any date is taken as the average. For the NBS 12(.), it is not clear whether there were measurement process problems or whether the block was

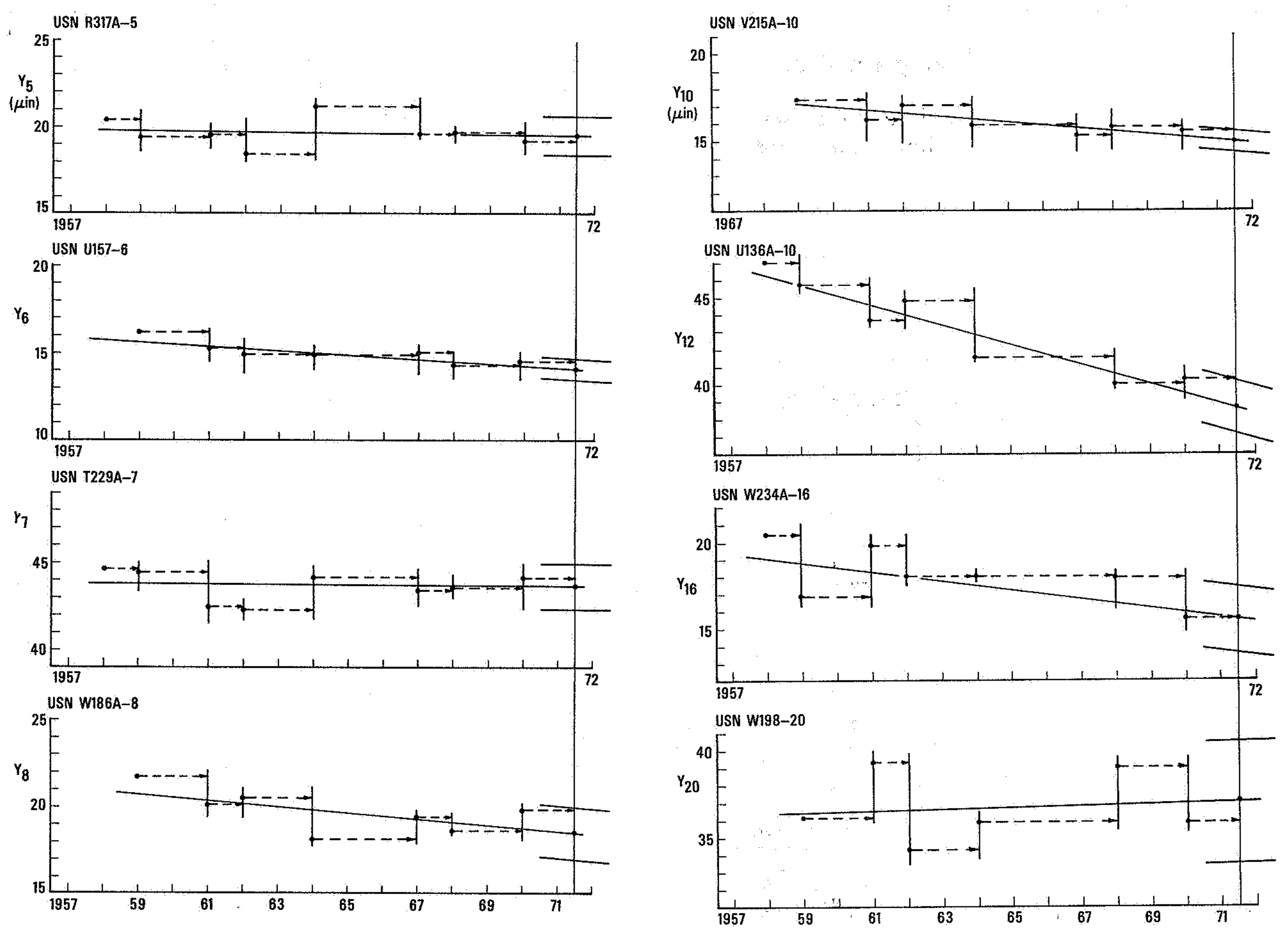


FIGURE 2. *Historical assigned values, USN.*

really changing. In this case, both the average and the fitted line were used to establish tentative values. In the case of NBS 20(.), only two points were available so that the estimate of K_1 is very weak. A summary of the predicted values and uncertainties for the date 1 July 1971 (7/1/71) is shown in table 2.

The uncertainty of the predicted value, as shown by the parallel lines above and below the point of intersection with the time line 7/1/71 in figures 1 and 2, is a function of the number of points in each collection, the degree of extrapolation beyond the time span encompassed by these points, and the standard deviation of the fit [11].

The uncertainty of the predicted value is computed by the relation

$$3\sigma_\beta = 3\sigma\sqrt{\frac{1}{n} + \frac{(t-\bar{t})^2}{\Sigma\,(t_i-\bar{t})^2}}$$

where n = the number of points in the collection;

t = time/date of the prediction;

$\bar{t}$ = average time/date (location of the centroid of time span covered by the measurement history, that is $\bar{t} = \Sigma t_i/n$);

t_i = time/date associated with each of the n values;

σ = process standard deviation (s.d. about the fitted line); and

σ_β = s.d. of predicted value at time t.

A summary of the predicted values and uncertainties, $(3\sigma_\beta)$, is shown in table 2. For each block, an estimated standard deviation, s, has been computed from the deviations of points from fitted lines shown. For both the NBS (.) and the USN reference blocks, s is plotted as a function of length in figures 3 and 4. The dashed line in figure 3 is an estimate of σ based on both sets of reference standards since the two sets are similar in all respects. The term σC, as defined in table 2, has been smoothed in figure 4 to obtain an estimate of σ_β for the NBS (.) group of blocks. σ_β for the USN blocks is estimated in figure 5. The uncertainties of the predicted values for the USN blocks is somewhat smaller than those associated with the predicted values of the NBS (.) blocks because the USN blocks were measured more frequently over the same time span.

TABLE 2
Summary of the Analysis of Historic Data

Block Ident.	7/01/71 Predicted Value β	No. Points n	s	$\hat{\sigma}$	C	$\hat{\sigma}C$	$\hat{\sigma}_\beta$	$3\sigma_\beta$
NBS(.)								
M136-5	29.5	5	.44	.62	.78	.48	.4	1.2
M115A-6	28.2	5	.76	.75	.78	.58	.52	1.6
M202A-7	15.4	4	1.63	.87	.33	.29	.64	1.9
M103A-8	47.2	5	1.25	1.0	–	–	.73	2.2
M109A-10	55.8	5	1.38	1.22	.78	.95	.95	2.8
M135A-12	68.8/76.0*	–	–	1.45	.33	.48	1.16	3.5
M109A-16	66.8	5	2.16	1.92	.80	1.53	1.59	4.8
A157-20	-0.2	2	–	2.4	–	–	2.02	6.1
USN								
R317A-5	19.5	8	.89	.62	.43	.27	.27	.8
U157A-6	14.1	7	.38	.75	.48	.37	.35	1.1
T229A-7	43.7	8	.95	.87	.43	.37	.4	1.2
W186A-8	18.5	7	1.04	1.0	.46	.46	.47	1.4
Y215A-10	14.9	7	.4	1.22	.46	.56	.6	1.8
U136A-12	38.7	7	1.0	1.45	.48	.7	.73	2.2
W234A-16	15.5	7	1.4	1.92	.48	.92	1.0	3.0
W198A-20	37.0	6	2.2	2.4	.53	1.3	1.26	3.8

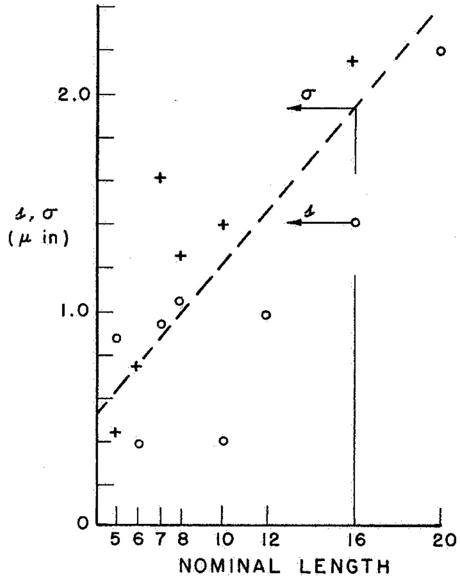
$\hat{\sigma}$ = estimated process standard deviation (figure 5)

$$C = \sqrt{\frac{1}{n} + \frac{(t_{7/1/71} - \bar{t})^2}{\Sigma(t_i - \bar{t})^2}}$$

$\hat{\sigma}_\beta$ = estimated standard deviation of predicted value (figures 6 and 7)

$3\sigma_\beta$ = estimated uncertainty of predicted value

* 68.8 based on average
76.0 based on computed rate of change



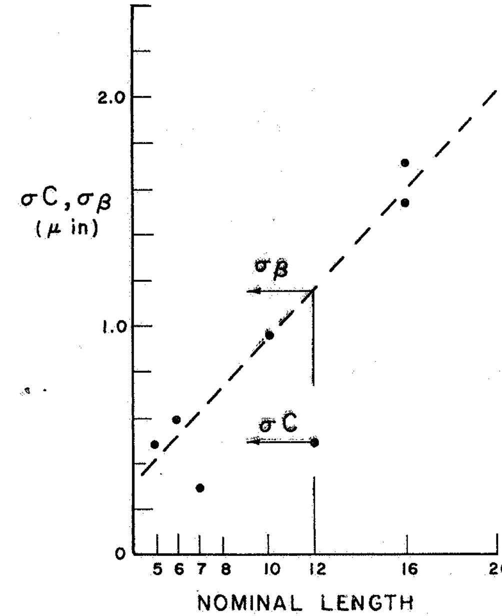
$$s = \sqrt{\frac{\Sigma(\text{dev.})^2}{n-2}}$$

o USN

\+ NBS (·)

σ = process standard deviation as estimated from s, deviation of points from fitted line.

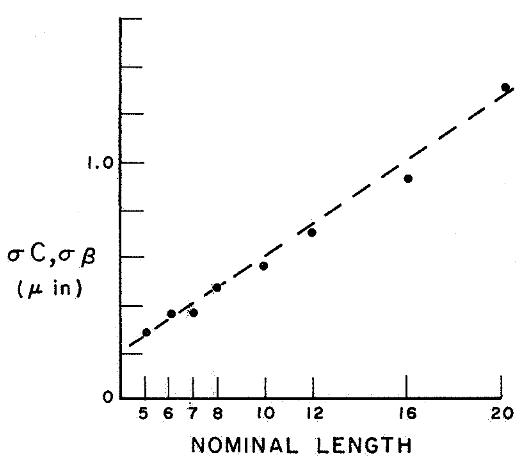
FIGURE 3. *Process standard deviation, historical.*



$$\sigma C = \sigma \sqrt{\frac{1}{n} + \frac{(t_{7/1/71} - \bar{t})^2}{\Sigma(t_i - \bar{t})^2}}$$

σ_β = Standard deviation of predicted value at time t = 7/1/71 (NBS (.) BLOCKS)

FIGURE 4. *S.D. of predicted value, NBS(.).*



$$\sigma C = \sigma \sqrt{\frac{1}{n} + \frac{(t_{7/1/71} - \bar{t})^2}{\Sigma(t_i - \bar{t})^2}}$$

σ_β = Standard deviation of predicted value at time t = 7/1/71 (USN BLOCKS)

FIGURE 5. *S.D. of predicted value, USN.*

4.2. Supporting Evidence

To verify the reasonableness of the uncertainty of the predicted values, an effort was made to review all of the measurement history relating to the NBS (.) blocks. In figure 6, as many independent value estimates as could be found or established are shown on the date the measurements were made. Where appropriate, the predicted value line and the predicted value for 7/1/71 together with the uncertainty limits of that value are shown. As before, all early data have been adjusted for changes in definition of the temperature scale and the inch.

The short horizontal line symbols represent independent values as determined by multiple wavelength interferometry, an independent measurement being defined as the length obtained for one wringing to an appropriate platen. The "reported" values shown in figure 1 are the average values from a collection of such independent measurements. Because the time lag between making the measurements and preparing the report was, in some cases, very long, the location

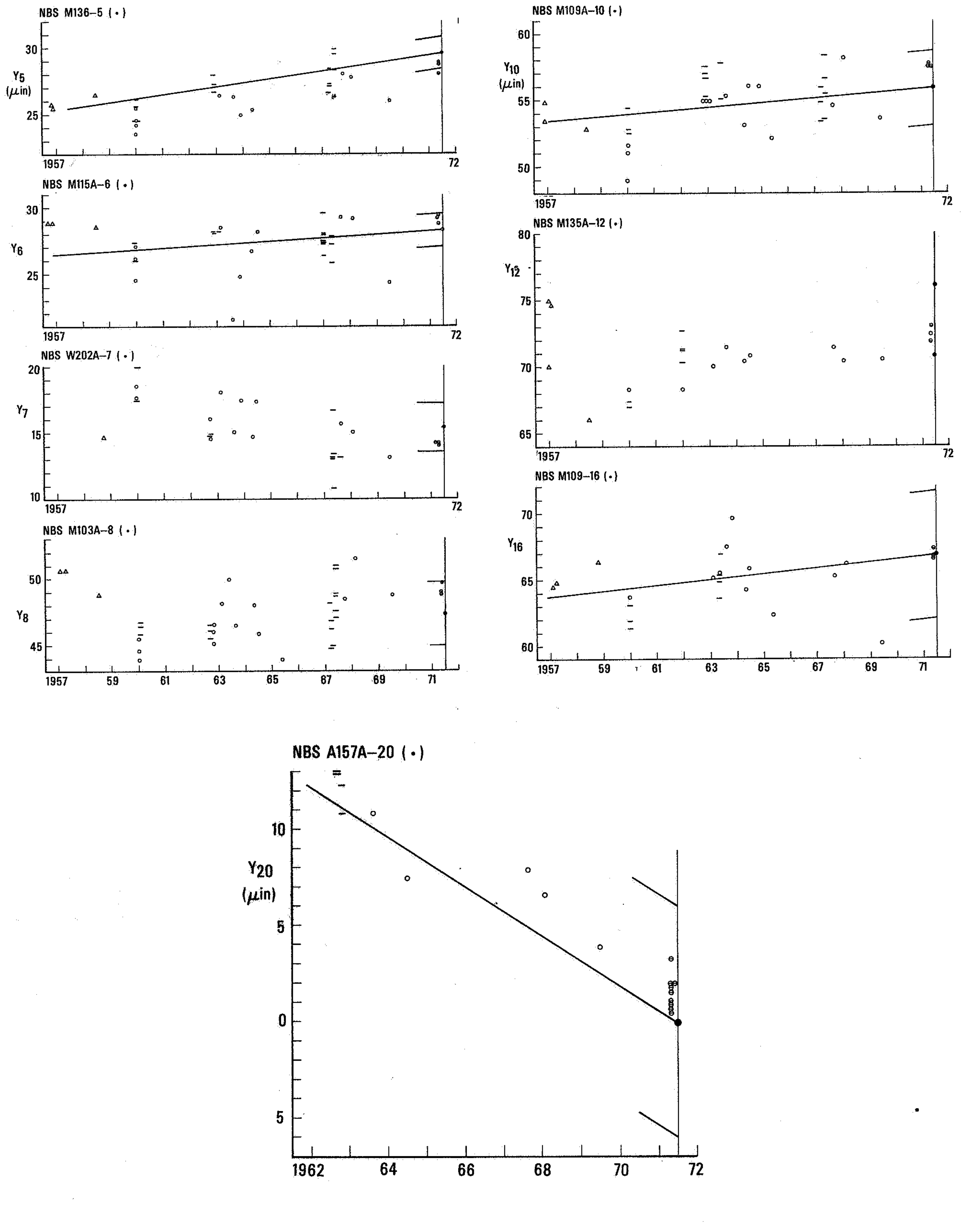


Figure 6. *Historical data, NBS(.).*

of the symbols on figure 6 reflect a realistic picture of process performance over time.

The results indicated by the triangular symbols on the left side of the figures are values relative to a line scale. Mr. B. Page made a careful comparison of the NBS (.) blocks with various intervals on a well known 40 inch line scale [12]. In this work the separation between scribed lines on each of two "cap" blocks was determined first with the "cap" blocks wrung together, then with the "cap" blocks wrung onto the ends of the gage blocks. The difference between the two separations was ascribed to be the length of the gage block. This was a difficult, tedious set of measurements, which, in effect related the gage block length to the traditional meter bar.

The most revealing information is indicated by the small open symbols. In all interferometric measurements which are made by the traditional methods,[8] the length, L, of the object is related to the difference in the length of two appropriate optical paths by the relation: $L=(N+f)(\lambda/2)$. In this relation N is an integer, f is a fraction, and the sum, $(N+f)$, is the difference in path lengths expressed in wavelengths. The term λ is the appropriate wavelength of the light source expressed in length units. The interferometer provides an estimate for the fringe fraction, f, thus for each object there are N possible solutions, only one of which is correct. Observing the same object with light sources of several different wavelengths (multiple wavelength interferometry) introduces some redundancy which can be used to substantially decrease the number of possible solutions. Even so, the separation between possible solutions is commensurate with the number of wavelengths available, and in most cases it is necessary to establish an estimate of the integral number of fringes by some other means. Prior to 1971, a mechanical comparison with "known" blocks was used to determine the approximate integral fringe order.[9] Since the redundancy of multiple wavelength interferometry could resolve over a range of several fringes, there were no stringent requirements on the mechanical comparison.[10]

As a consequence of the above procedure, for each set of gage blocks which was calibrated by the old interferometric process, the data included a set of mechanical difference measurements with respect to the NBS (.) blocks. Accepting the interferometric values as the best estimates of the lengths of the blocks under test, these values, together with the mechanical difference measurements, provide, in essence, "new values" for the NBS (.) blocks, as shown by the small open circle symbols. In all cases, these "feedback" values do not deviate from the fitted lines, or average value where appropriate, in excess of the most optimistic uncertainty estimates previously listed. This evidence suggests that the long, tedious interferometric measurements did little more than "verify" that values as transferred from the NBS (.) blocks were appropriate. This evidence also suggests that the inherent precision and simplicity of the mechanical comparison process was being ignored.

The small circular symbols with the horizontal lines, immediately to the left of the "7/1/71" time lines in figure 6 provide additional evidence to support the routine use of a mechanical transfer process. Because of the good agreement between the historical values and the "feedback" values as described above, formal mechanical comparisons were made, following the procedures described in section 5.3. The values indicated by the Θ symbols are the values for the NBS (.) blocks relative to the USN historical predicted values. This work is summarized in table 3.

4.3. Establishing "Old" Accepted Interferometric Values

The supporting evidence for the NBS 12(.) seems to indicate that the proper value is between the average value and the predicted value based on the estimated slope. In all other cases, the supporting evidence seems to verify the predicted values. It is of interest to note that values established by one-to-one mechanical comparisons used in the old interferometric process, by a defined single interferometric measurement, by the "cap block" method with reference to a line scale, and by the more sophisticated intercomparison designs, appear to fall within limits which are not at all unreasonable with reference to the uncertainty of the predicted value.

The assignment of accepted "old" interferometric values is shown in table 4. With minor changes, as explained, these are essentially the same as the tentative values of table 2. The estimates of the coefficient K_1, the rate of change of length in microinches per year, are computed in table 5. These estimates will be subject to considerable discussion, and some revision, later in this paper.

Finally, the NBS (.) blocks and the USN blocks have been used in pairs in many mechanical comparisons. One output of the design is the difference as measured mechanically, between the NBS (.)

[8] Two methods for using interferometry are described in some detail in appendix 3. The traditional method, sometimes called static interferometry, or multiple wavelength interferometry, has normally been used to assign length values to selected objects by metrology laboratories. This method should not be confused with "fringe counting" interferometry, in which one element of the interferometer is moved along the interval of interest.

[9] The mechanical comparison process is described in detail in appendix 4.

[10] In the past, a value assigned on the basis of mechanical comparison was determined by simply comparing the "unknown" with one or more "knowns." With the exception of values referred to in this paragraph, all other comparison values in this paper are based on comparison designs discussed in Section 5.

TABLE 3

Summary of USN and NBS(.) Predicted Values and NBS(.) Values Assigned Relative to USN Value by Mechanical Comparison

Nominal Size (in)*	USN Predicted Value Y(I)	Uncertainty	NBS(.) Relative to USN**	NBS(.) Predicted Value Y(I)	Δ	Uncertainty NBS Predicted Value
5	19.5	.8	29.1	29.5	− .4	1.2
			28.3		−1.2	
			29.0		− .5	
6	14.1	1.1	28.8	28.2	+ .6	1.6
			28.7		+ .5	
			28.3		+ .1	
7	43.7	1.2	15.5	15.4	+ .1	1.9
			14.6		− .8	
			14.6		− .8	
8	18.5	1.4	48.4	47.2	+1.2	2.2
			47.6		+ .4	
			47.7		+ .5	
10	14.9	1.8	57.1	55.8	+1.3	2.8
			56.9		+1.1	
			56.9		+1.1	
12	38.7	2.2	72.4	68.8/76.0	−	3.5
			70.8		−	
			70.3		−	
16	15.5	3.0	67.2	66.8	+ .4	4.8
			66.8		0	
			66.6		− .2	
20	37.0	3.8	4.0	−.2	4.2	6.1
			3.7		3.9	
			1.7		1.9	
			2.5		2.7	
			1.3		1.5	
			1.8		2.0	
			2.7		2.9	
			2.7		2.9	
			2.3		2.5	

* Except for nominal size, all values in μin

** 3 degrees of freedom in each measurement

TABLE 4

NBS(.) ACCEPTED PREDICTED VALUES BASED ON HISTORICAL DATA AND MECHANICAL COMPARISONS

Nominal Size	NBS(.) Accepted Value Y(I)	Estimated Uncertainty	Source
5	29.5	1.2	Table 2
6	28.3	1.6	Table 2
7	14.9	1.9	Average NBS Relative to USN - Table 3
8	47.9	2.2	Average NBS Relative to USN - Table 3
10	56.0	2.8	Minor Adjustment Based on Difference Measurements (55.8 from Table 2, 51.0 Average from Table 3)
12	70.8	3.5	Minor Adjustment Based on Difference Measurements (Uncertainty from Table 2, 71.2 Average from Table 3)
16	66.8	4.8	Table 2
20	1.6	6.1	Minor Adjustment Based on Difference Measurements (−.2 from Table 2, 2.5 Average from Table 3)

TABLE 5

Time-Rate of Change of Length

Nominal Size	Block Ident.	Values from Graphs 1/1/71	1/1/58	1/1/62	Difference (μin) '71-'58	Difference (μin) '71-'62	No. of Days	K_1 (Slope μin/yr.)
	NBS(.)							
5	M136	29.5	25.6		3.9		4748	.300
6	M115A	28.2	26.1		2.1		4748	.162
7	W202A*	-	-		-		-	0
8	M103A*	-	-		-		-	0
10	M109A	55.8	53.5		2.3		4748	.177
12	M135A*	-	-		-		-	0
16	M109A	66.9	64.0		2.8		4748	.215
20	A157	-.2	-	12.3	-	-12.5	3287	-1.478

* Data does not indicate conclusively changes of length with time. K_1 is assumed to be zero for these blocks.

TABLE 6

(.) - USN (Typical)

Nominal Size	Computed Diff. Between Predicted Values	Measured Diff. by Comparison	Δ	Estimated Uncertainty of Computed Diff.
5	7.4	9.0	1.6	1.6
6	15.2	14.4	-0.8	1.7
7	-28.5	-28.9	-0.4	2.2
8	29.7	29.4	-0.3	2.4
10	41.0	41.8	+0.8	2.6
12	31.6	31.8	0.2	2.9
16	50.2	50.2	0	3.5
20	-36.7	-35.2	1.5	4.2

Δ = [(Measured Diff. by Comparison) - (Computed Diff. Between Predicted Values)]

and USN block. These differences should agree with the difference between the accepted "old" interferometric values. While this point will be discussed in detail later, a typical comparison of the computed difference between the predicted values and the measured difference as determined in comparisons is given in table 6.

5. A New Point of Departure

5.1. Definitions

Two noncoincident terminators along a specified coordinate axis determine a length interval. Three such intervals are of interest. For the defined unit, the interval is the wavelength of a specified radiation, the terminators being defined by interferometry, and the coordinate axis being defined by the axis of the interferometer. Practical access to this unit is through artifacts typified by the line scale and the gage block. For the line scale, the terminators are lines marked on a reasonably flat surface. The coordinate axis is usually defined relative to some additional markings on the scale surface. For the gage block, the terminators and the coordinate axis are related to the geometric form of the block. The length interval embodied in both types of artifacts must be related to the defined unit with error limits compatible with the manner in which the artifacts are to be used.

In theory, relating the line scale to the defined unit is a simple displacement measurement. A suitable detector initially centered on one terminator can be moved along a parallel coordinate axis to a position centered on the other terminator. The movement, or displacement, can be measured in terms of the wavelength of some suitable light source, and this in turn is assigned to be the length of the interval defined on the face of the artifact. This is a symmetrical measurement in that the detector can approach the terminators from either direction, searching, if necessary, for some reproducible "center." In practice, such a measurement over a long interval is difficult primarily because of the lack of rigidity of both the artifact and the measuring equipment.

In the case of the gage block, terminators and the coordinate axis are not precisely defined by the geometry of the block. The fact that the terminators are on opposite faces of the block with the coordinate axis going through the block means that the sensing device can only approach a terminator from one direction, and as a consequence, no sensing device can approach both terminators from the same direction. Such a measurement is called a separation measurement. For measurements of this type, the measurement process must be defined in such a way that the sensing device can approach both terminators from the same direction, and the nonsymmetry of the sensing device relative to the terminator must be considered.

Traditionally, gage blocks are made in ordered sizes so that they can be assembled in stacks to create a variety of artifact lengths. This suggests that the desired length could be considered to be the separation between two parallel planes, one being the surface of the block, and the other being the surface to which it is mated. Wringing the block to a suitably large flat surface not only simulates one usage, but also establishes a terminator surface which can be approached from the same direction as the terminator surface on the opposite end of the block. If both terminator surfaces, that is the block and the flat or platen, have very nearly the same optical properties, the problems introduced by lack of symmetry will be minimized.

The degree of flatness of the two terminator surfaces, and the degree to which the two surfaces are parallel are manufacturing value judgments. Descriptors for "out of flat" and "out of parallel" involving non-flat surfaces are, at best, semi-quantitative. (The traditional method of determining these descriptors is given in reference [13].) For the purpose of measurement, a specific terminator, or "gaging point," is designated on the visible, or "top," surface of the block. The assigned length is the separation between a point and a plane, the point being the defined "gaging point," and the plane being a suitable platen surface which is in close proximity, that is "wrung," to the bottom surface of the block. In order to assign a length value to an object such as a gage block, the surfaces must be sufficiently flat to produce an interferogram which can be interpreted and to obtain an acceptable "wring" with similar objects and appropriate platens.

The length assigned to a block or an object at some time, t_o, and at some temperature, T_o, can be expressed in two ways as shown for a 5 in block:

Block length

$$= L_m(t_o, T_o \qquad = 4.999\,975 \text{ in}$$

$$= (Nom + Y_m(t_o, T_o) \qquad = (5 - 0.000\,025)\text{ in}$$

where the subscript m designates the measurement process used to determine the value, and "*Nom*" designates the nominal size of the object. (In this paper m is assigned the symbol H for values determined by the historical multiple wavelength interferometric process, as discussed in section 4.1.; I for the new single wavelength interferometric process to be described in section 5.2.; and II for the transfer of values by mechanical comparison as described in section 5.3.)

The past practice had been to limit "*Nom*" to certain selected numbers. With $Nom - Y_m(t_o, T_o$ less than ϵ, an arbitrarily small tolerance limit, in many practical uses $Y_m\ (t_o, T_o)$ is assumed to be zero. Also, the coefficient of change with time had been eliminated by discarding blocks which show change. The temperature at the time of comparison has been controlled so that T is very nearly equal to T_o. The present intent is to emphasize $\hat{L}_m(t,T)$ without restriction on the magnitude of $\hat{Y}$, time, temperature or material.

The block length at any time, t, and at any temperature, T, can be expressed by the relation:

$$L_m(t,T) = Lm(t_o, T_o) + K_1(t - t_o)\ (1 + K_2(T - T_o)) \pm (3\sigma + S.E.)$$

where K_1 is the natural rate of change of length with time, K_2 is the thermal coefficient of expansion, σ is the appropriate estimator of the random variability of process m, and S.E. is the appropriate systematic error estimate for process m. Each term in this relation is discussed in detail in appendix 1.

The parameters in the above formula cannot all be determined in a single sequence of measurements. K_1 must be estimated from historical data. The formula shown assumes the uncertainty of K_2, the coefficient of thermal expansion, to be small relative to the precision of the process. The numerical value of K_2 currently used, as in the past, is appropriate only because laboratory conditions are held very close to 20 °C, the accepted temperature, for reporting lengths. In terms of usage, however, the formula should be valid over a temperature range of at least 5 °C. To achieve this, some future efforts must be directed toward establishing block transient thermal characteristics, block temperature and appropriate coefficients of expansion.

With the present state of the interferometer technology, there is no way to introduce a redundancy into the measurements other than straightforward repetition. Each interferometric measurement produces an independent estimate of a length relative to a wavelength scale. Under this condition the amount of work necessary to establish realistic estimates of uncertainty of the values, as evidenced by this paper, makes it economically impossible to consider such measurements as routine. A more practical means must be used to transfer the unit from the defining wavelength to the point of use.

The procedural operations in the new measurement process are, in essence, merely refinements of earlier techniques differing mainly in the order in which they are performed. (See fig. 7.) Initial value assignment to reference standard blocks is the output of an interferometric measurement process. The transfer of a length unit as represented by this assigned value to another block is based on direct comparison which at the present is done with contacting comparators. This sequence has been chosen because of the relative standard deviations of the two processes, that of the interferometric process being somewhat larger than that of the mechanical process. In order to not degrade the transfer in the comparative processes, the new NBS interferometric process is limited to the measurement of NBS reference standards and other similar objects which support the total length measuring system.

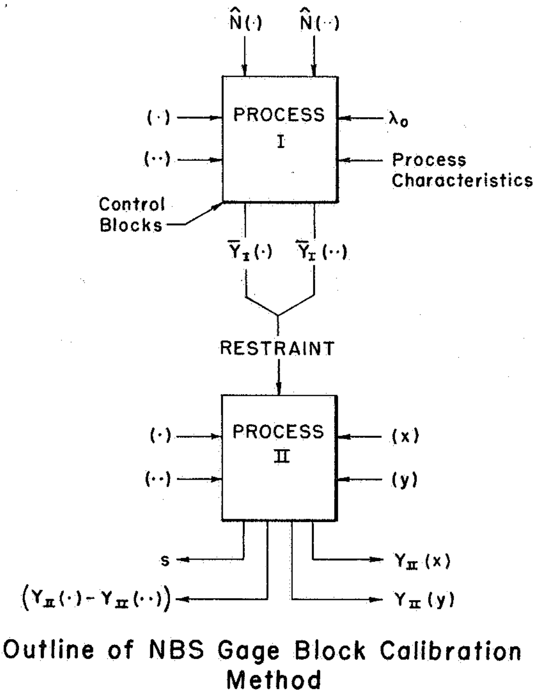


FIGURE 7. *Outline NBS gage block calibration.*

Process I is a single wavelength interferometric measurement process. The inputs to Process I are: the reference blocks, that is the (.) and (..) blocks; a light source of known wavelength λ_o; and the length of the reference blocks expressed in integral "fringes", $\hat{N}(.)$ and $\hat{N}(..)$. The process outputs are estimates of length expressed as corrections to a nominal value, $Y_{\mathrm{I}}(.)$ and $Y_{\mathrm{I}}(..)$, normalized to a temperature of 20 °C. The state of control of the measurement process is established partly by collections of measurements of the (.) and (..) reference blocks, and partly by measurements on selected "control" blocks which have been chosen to emphasize certain types of systematic errors which might be present in the results.

Process II is a transfer process based on differences as determined by comparison. The reference blocks, (.) and (..), their accepted values, $Y_{\mathrm{I}}(.)$ and $Y_{\mathrm{I}}(..)$, and the associated uncertainties, together with the "unknown" blocks (x) and (y) which may or may not be similar to (.) and (..), are the inputs to Process II. One output is the values for the "unknown", $Y_{\mathrm{II}}(x)$ and $Y_{\mathrm{II}}(y)$ at temperature 20 °C, together with appropriate uncertainties. Control outputs are an estimate of the process standard deviation, s, which will eventually establish an accepted process standard deviation, σ, and a measured difference between the reference blocks, $(Y_{\mathrm{II}}(.) - (Y_{\mathrm{II}}(..))$. It must be demonstrated that Process II will operate in a state of control, and that the characteristics of like processes are similar. It must be verified that the measured differences determined by Process II agree with the computed differences obtained from Process I, within the uncertainty of both processes.

5.2. Single Wavelength Interferometry

One shortcoming associated with all static interferometric measurement processes (see appendix 3) is that one can only observe fractional fringe differences between the fringe patterns associated with the top of the block, and the platen to which it is "wrung." The integral number of fringes associated with the fraction, to express the length of the block or artifact, must be established by other means. Initially, one relied on the skill of a master craftsman to construct a set of blocks such that the deviation from selected nominal values was very small. The redundancy of multiple wavelength interferometry provided a means to resolve these small differences. For longer lengths, or differences, one was faced with the problem of multiple solutions, the length equivalent between solutions generally increasing with the number of different wavelengths used. In general the accepted solution was the one which was closest to some initial estimate of the length, or difference, provided the uncertainty of this estimate was considerably less than the interval between possible solutions. To assign values to long blocks by these methods to the level of uncertainty historically stated was a monumental task. In effect the measurement was a stepping process involving a sequence of blocks having length differences commensurate with the coherence of the available light sources.

The coherence of stabilized laser light sources removes previous limitations on the length of the optical path, thus simplifying the interferometric measurement of long blocks. The stabilized laser is also easy to operate and the intensity of the light beam, even with small apertures, makes the photographing of "fringe patterns" practical. The assignment of the wavelength to the laser radiation can be made with reference to the defining radiation of Krypton, or some other suitable well characterized radiation. The details of the interferometric process as currently used are described in reference [14].

For computational purposes, a single measurement for Process I is defined by the following steps:

(1) Wring block to appropriate platen.

(2) Photograph fringe pattern.

(3) Compute the length from the average of four independent photo interpretations for the fraction f, and the effective λ.

(4) Several hours later repeat steps (2) and (3).

(5) The single measurement is the average of the values computed in steps (3) and (4).

The complexity of the process is such that it cannot be readily modified to utilize the power of intercomparison designs as a means of monitoring the state of control. The required redundancy, however, can be obtained from repeated measurement of the same set of objects. Control blocks, or "check standards," in the form of three gage blocks permanently wrung to suitable platens were chosen. Repeated measurements of the three blocks, at frequent but random intervals, provided sequences of data suitable for monitoring the process. The "check standards" are not removed from the platens between measurements, therefore, the variability introduced by the wringing process is not reflected in the control data.

The chosen "check standards" consist of a 0.150 in chrome-carbide block, a 10 in summation of cervit blocks (4 in + 4 in + 2 in), and a 10 in steel block. In interpreting the accumulated data, the variability associated with the long steel block should approximate the variability associated with the normal process output, except for wringing film variability. The variability of the cervit block should reflect length dependent variability not associated with temperature. The variability of the chrome-carbide block should represent a sort of ultimate performance. Repeated measurement on other objects, such as the reference blocks, should provide data for evaluating the effects of wringing.

The initial work with the interferometric process described above was to establish values for the NBS(.) and NBS(. .) groups of reference blocks. For the (.) group, the "tentative" values were taken from table 4 in section 4.3, and for the (. .) group, from table 14 in section 6.1. This work is summarized in table 7. A tentative estimate of the uncertainty of a single value was computed using successive differences, as shown in table 7 and figure 8. (This initial estimate of process precision, $s = 0.635$ μ in, will be revised later in this paper as additional measurements are made.)

In the course of the above work, frequent measurements of the control blocks were made at random times. The results of some of these measurements are shown in figure 9. The initial wide variability reflects in part the performance of the interferometric process during the measurements of the (.) and (. .) blocks. Analysis of this data provided guidance toward improving the process precision. (See sec. 7.1.) After process improvement, the results of measurements on the three control blocks appear well behaved. The details of this improvement are discussed in reference [14]. It is of interest to note that in the course of obtain-

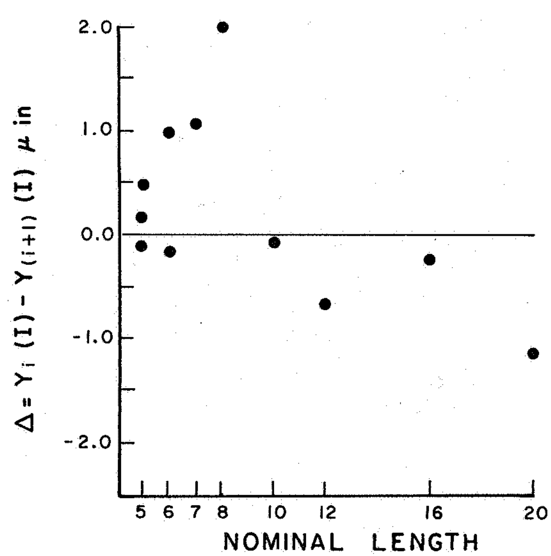
TABLE 7
Summary New Interferometric Process Data
(PROCESS I, (January 1972)

Block Ident.	1/01/73 β μin	n	$X_i - X_{i+1}$ μin	$(X_i - X_{i+1})^2$	$\frac{3\hat{s}}{\sqrt{n}}$ μin
NBS(. .)					
H178-5	0.9	3	.15	.02	1.1
			.45	.2	
H312-6	−2.1	2	.95	.9	1.35
H105-7	−5.8	2	1.05	1.1	1.35
H143-8	7.4	2	1.95	3.2	1.35
H148-10	−1.4	2	−.1	.01	1.35
H249-12	14.9	2	−.7	.49	1.35
H155-16	10.3	2	−.25	.06	1.35
H146-20	21.7	2	−1.2	1.44	1.35
NBS(.)					
M136-5	29.3	2	−.1	.01	1.35
M115A-6	29.4	2	−.2	.04	1.35
M202A-7	15.5	1	−	−	1.9
M103A-8	50.7	1	−	−	1.9
M109A-10	58.4	1	−	−	1.9
M135A-12	71.9	1	−	−	1.9
M109A-16	67.7	1	−	−	1.9
A157-20	1.1	1	−	−	1.9
				Σ = 8.07	

$$\hat{s} = \sqrt{\frac{8.07}{2\text{x}11}} = .635$$

SUCCESSIVE DIFFERENCES

$$S = \sqrt{\frac{\Delta^2}{2n}} \approx 0.635 \, \mu \text{ in}$$



Estimate of interferometric process precision by successive differences.

FIGURE 8. *Interferometric process precision by successive differences.*

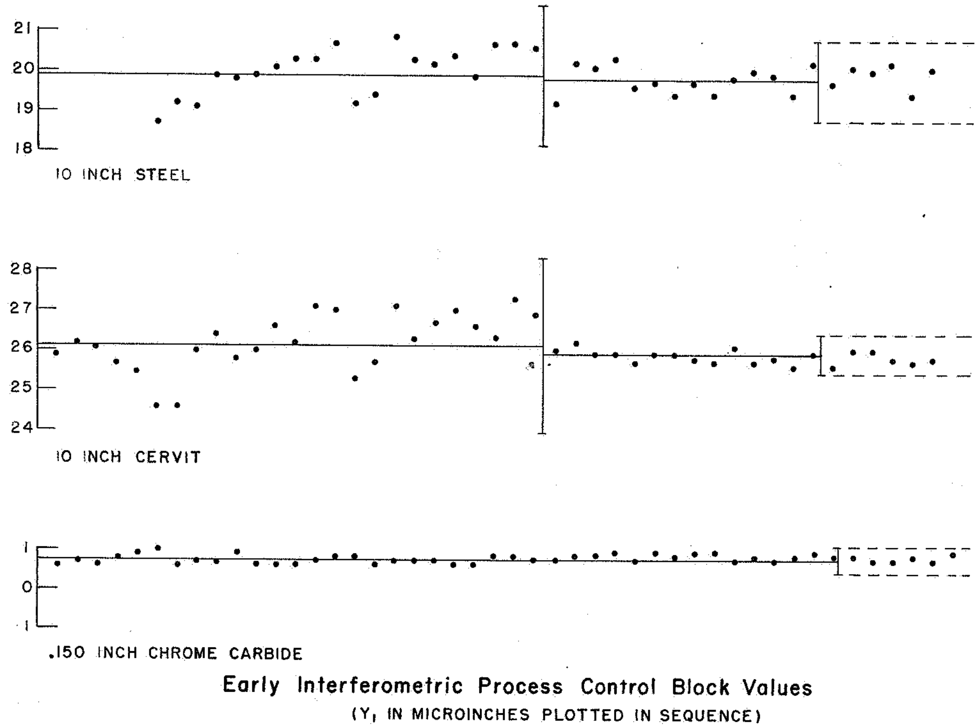


FIGURE 9. *Early interferometric process control block data.*

ing improved performance, the change in average values for the control blocks was insignificant. Future measurements will reflect the benefits of the changes made.

5.3. The Comparison Process

At the present time, by far the largest usage of long gage blocks is associated with various types of contacting comparators such as described in appendix 4. The use of these comparators provides a precision of measurement compatible with most requirements. The precision of some comparative processes can be substantially smaller than the precision of the interferometric process. A measurement process designed around this type of instrument will not only provide a means to transfer the unit, but will also provide guidance to similar processes in which the transfer standards are used to measure other objects.

In formulating a measurement process one must define (1) the manner in which the instrument indication is related to the measurement unit, and (2) the sequence of operations which are to be used for a "single measurement." The announced, or reported, result can be from a "single measurement" or the average of several "single measurements." For groups of similar objects, an intercomparison design provides a redundancy which is more efficient than a specified number of "single measurements" for each of the objects in the group. For example, to obtain a result which is the average of 4 "single measurements" for each of two "unknowns" with reference to two "knowns" would require 16 "single measurements." With the use of a comparison design, the same redundancy can be obtained with 6 or 8 "single measurements", depending on how the design is formulated. In addition, other desirable features can be incorporated into the design.

Initial studies were made using a comparison sequence designated *ABBA* (sometimes called a double substitution comparison). In this sequence, object *A* is inserted in the comparator producing an observation 0_1; object *B* is inserted to produce 0_2; and then the sequence is repeated in reverse order. This can be illustrated as:

Observation	*Separation Interval*
0_1	$S-A$
0_2	$S-B+d$
0_3	$S-B+2d$
0_4	$S-A+3d$

where S is the unknown separation of the comparator head and anvil and d is an assumed uniform incremental drift occurring in the interval between the observations. If the sequence is made on a reasonably uniform time scale, linear drifts are eliminated in computing the estimate:

$$(A-B)=K(0_1+0_4-0_2-0_3)/2$$

The differences in instrument indication in the above relations are converted to length units by the constant K. In most cases, the instrument is adjusted before making the comparisons so that $K\approx 1$, and, as a consequence, the observed differences are assumed to be expressed in microinches. The repetition of the difference measurement in reverse order eliminates the effects of linear drifts, or trends, d, in the announced difference, $(A-B)$.

The above set of observation equations can be solved for average linear drift or trend:

$$d=\frac{1}{10}\,(-30_1-0_2+0_3+30_4)$$

While the *ABBA* "single measurement" incorporates trend elimination, the design described in appendix 2 does not. After some experiences with this combination, because of short time intervals required to make the necessary comparisons, it was felt that trend elemination should be incorporated in the design. A trend elimination design involving 8 measurements was adopted.[11]

The use of this design resulted in a decrease in standard deviation for the 12, 16 and 20 in blocks, therefore, the "trend elimination" design was used for the work described in this paper.

An analysis of many series of measurements indicated that the computed linear drift, d, was essentially zero. This was ascribed to the stability of the comparators, and the short amount of time required to make the required comparisons. The comparison repeated in reverse order appeared to be a wasted effort. As a consequence, the "single" measurement is now defined as *AB* (sometimes called a single substitution comparison), where:

$$A=S+0_1$$
$$B=S+0_2+d$$

where d for all practical purposes is zero, and:

$$(A-B)\approx K(0_1-0_2)$$

Currently (Since January 1974) the gage block measurements are made with the AB "single measurement" and the following design which is somewhat more efficient than the design shown above:[12]

	(.)		(x)	(y)
$A(1)$	+			
$A(2)$				+
$A(3)$			+	−
$A(4)$		+		
$A(5)$		+		−
$A(6)$	−			+
$A(7)$	+			
$A(8)$				
R	+	+		
C	+			

This design is described in detail in reference [6].

Generally speaking, the current requirement that all items being compared have the same nominal length is due to the limited on-scale range of the comparators and not a limitation of the comparison design. These, or similar, designs can be used with large "on-scale" range comparators now under construction, with minor changes in the definition of a "single" measurement for comparators with "on-scale" ranges of several inches.

5.4. Transfer Techniques

Developing a measurement technique is, to a certain extent, trial and error. Initially, a sequence of operations is established based on one's best judgment. Minor procedural changes may be necessary to achieve the desired results—a sequence of repeated measurements which tend to cluster about a limiting mean and are free of identifiable trends, groupings or abrupt changes. The distribution of the initial collection of values is a characteristic of the process. In most cases, an adequate descriptor of this distribution is the standard deviation. The standard deviation of the initial collection becomes a "yardstick" against which one can assess the effect of changes in the sequence of operations. Introducing changes one at a time will give sequences of results in which the standard deviation is either significantly larger than, about the same as, or significantly smaller than the initial standard deviation. By appropriate action, one finally determines a detailed procedure which produces results within acceptable limits and free from correlation with all known sources of variability. At this point predictive limits, within which it is almost certain that the next measurement result will fall, can be based on actual process performance. Each new measurement verifies the validity of these limits.

[11] The design used (later replaced) was as follows:

	(.)	(. .)	(x)	(y)
$A(1)$				
$A(2)$				
$A(3)$				
$A(4)$				
$A(5)$				
$A(6)$				
$A(7)$		−		
$A(8)$		+		
R		+		
C		−		

[12] The expected variability of the result from a design is a function of the standard deviation of a "single measurement" and the redundancy incorporated in the design. If the standard deviation of a "single measurement" is σ, for the design shown, the standard deviation of the value of one unknown is 0.52σ. A comparable standard deviation for the previous design is approximately 0.58σ.

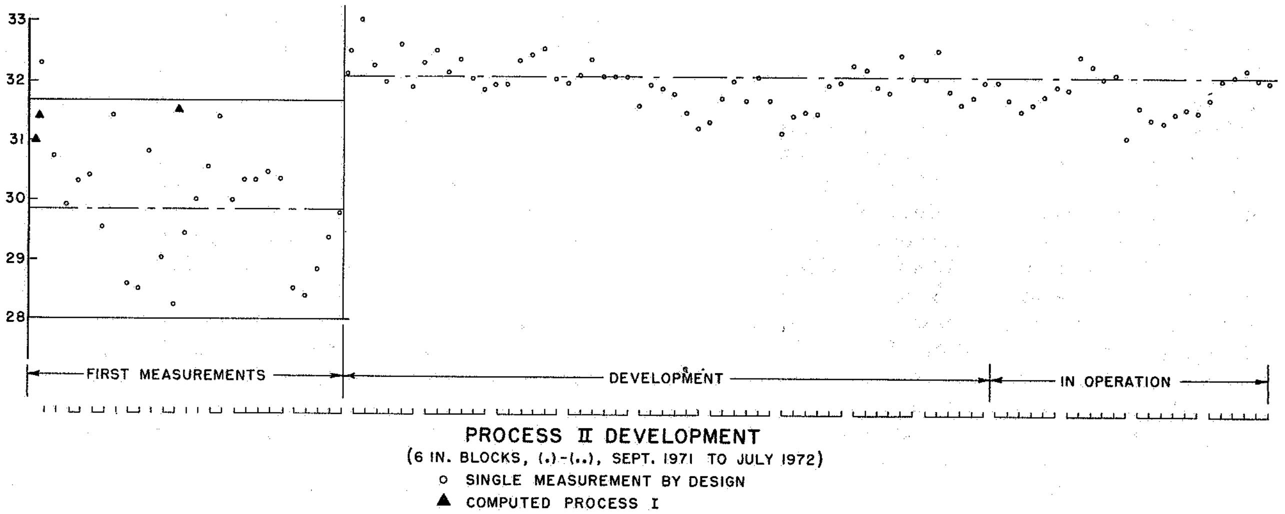


FIGURE 10. *Process development.* (6(.)–6(. .)).

A typical sequence of results during the process formulation period is shown in figure 10. The measurements were made in a slightly pressurized, temperature controlled room in which the level of illumination is held quite low. (While such facilities are not unique to NBS, the environment is restricted and may be significantly different than that in which other facilities must work with long blocks.) Many things were tried between the first measurements and the operational measurements, not all of which were significant in terms of improved performance. Clearly, in the beginning (from November 9, 1971 until March 22, 1972) the differences between the two "known" or restraint blocks as determined in the comparative process were widely scattered and offset from the expected difference as determined from the accepted interferometric values.

Having achieved a performance in which the process appears to be well-behaved with a mean difference at least in the neighborhood of the expected difference, one can proceed with the process development. A frequently overlooked step is to purposely change procedures in an attempt to degrade the process performance. This technique will clearly identify the significant elements of the process. Factors studied in the development interval shown include methods of placing the block in the comparator, agreement between operators, time interval allowed for temperature equilibrium, and location of comparator indicating system. In addition, the contact pressure and "readout" calibration were checked at frequent intervals.[13] It was found that once the blocks had initially reached a state of temperature equilibrium with the environment, time intervals between sequences of measurements could be reduced from in excess of 4 hours to about 30 min without degrading the result. The stability of the process is demonstrated by its

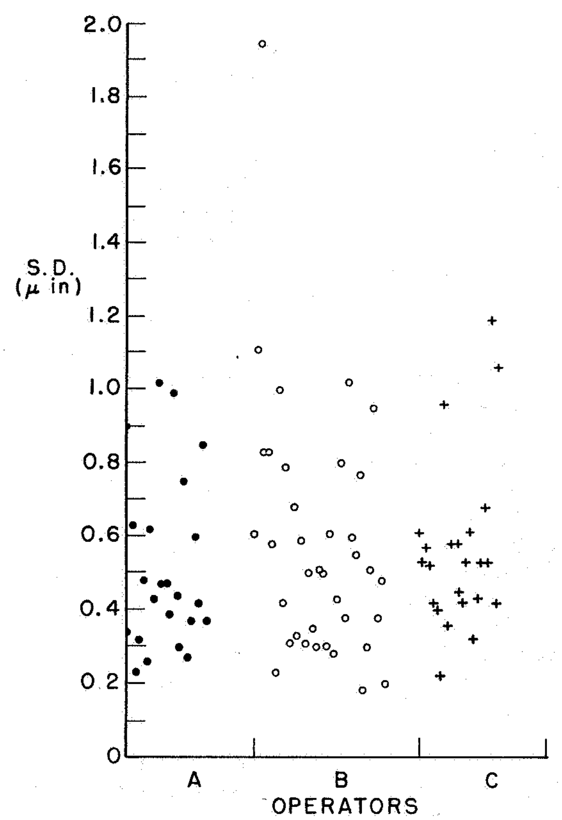


FIGURE 11. *Operator standard deviation.*

[13] Reference [15] describes the procedures necessary to verify that the comparator is operating properly. Reference [16] describes the procedural detail now used in the NBS comparison process.

performance through the development stage, as shown in figure 10, since many things were tried between the first measurements and the operational measurements.

One essential requirement is that all operators must be able to follow the procedures and produce consistent results. Figure 11 illustrates the standard deviations obtained by three different operators for sequences of like measurement. In the end, it was decided that careful block handling, attention to cleaning details, and a light stoning of the surfaces of the block and comparator anvil were essential elements in achieving a state of control.

Process simplification is a continuing task. One is constantly searching for methods or procedures which conserve measurement effort without degrading the process performance. There is no "one way" to carry out the detailed procedures which would be applicable to all measurement processes. On the other hand, the detailed procedures used in a process are a part of the defined measurement. Ideas from other measurement processes can be adopted after there is verification that such procedures do not degrade the performance of one's own process.

5.5. Thermal Conditions

It has long been suspected that the largest source of variability in the measurement of long blocks has been thermal effects. Errors from this source can enter the measurement in two ways, as a systematic effect or offset, and as a source of randomlike variability. For example, the temperature of the sensor may be offset from the average temperature of the block. The phase and magnitude of temperature variability at the sensor may also differ from changes occurring in the temperature of the block. It is most important that blocks be at very nearly the same temperature at the time of comparison. Equalization must occur in a uniform temperature environment (no significant horizontal temperature gradient).

Block temperature changes occur because of both conduction and radiation. For example, a warm block is usually placed on a cool surface plate, in an airstream, to come into temperature equilibrium with the measurement environment. Most of the initial temperature change is probably a result of conduction from the block to the air and the surface plate. Some is due to radiation. Having reached thermal equilibrium with a particular environment, the same block, when exposed to a heat source such as a light or operator in close proximity, will immediately start to change in temperature. Such changes are largely associated with radiation.

The systematic effects are usually minimized by making all measurements in a controlled temperature environment. The base temperature can be set sufficiently close to the accepted reporting temperature, 20 °C, so that the uncertainty in the thermal coefficient of expansion of the block material is not significant. It is recognized that this practice restricts the usefulness of the number assigned to the blocks.[14] One of the ongoing programs is the development of procedures to verify, or establish if necessary, coefficients of expansion for each block sufficient for use over a temperature range of 20 to 25 °C with minimum degradation of uncertainty of the computed values for temperatures in this range.

Blocks are normally stored on the instrument platen to permit temperature equilibration, and then handled with special tongs when making the comparisons. The comparators are partially enclosed with barriers made from insulating material to minimize the effects of horizontal thermal gradients. From the start, reflective coated mylar smocks have been worn by the operators to minimize radiation effects. The level of illumination in the measurement laboratory is quite low, and as a consequence little attention was given to the finish on the nongaging surfaces of the blocks.

The agreement between the results of measurements repeated in various laboratories is a part of process development. In one early study, the differences between two 16-inch blocks as determined by NBS and by a cooperating laboratory were not in agreement. This is shown by the data from September 6, 1970 to April 3, 1971 in figure 12. Many things were checked to determine a plausible explanation, it finally being decided that the discrepancy might be associated with the markedly different finish on the nongaging surfaces of the two blocks (one was bright and the other was dull and mattelike), and the difference in illumination in the two laboratories (one was practically dark and the other a well lighted general purpose lab). If one assumes that a steady state block temperature is based on equilibrium in heat flow to and from the block, all other things being equal, the differential block temperature would not be the same under the two conditions, the lighted lab and the dark lab.[15]

Between April 3, 1971 and August 23, 1972, studies were made on the thermal response of blocks subjected to a radiant energy shock (in the form of turning on and off a fluorescent light located a few feet from the block while measuring the block temperature). It was found that a wrapping of two (or more) layers of gold-coated mylar film essentially made test blocks with different finish on the nongaging surfaces appear to have uniform thermal

[14] A continuing question with regard to long blocks concerns the ability to demonstrate closure at temperatures other than 20 °C. At the present time it is assumed that the linear coefficient of expansion of the (.) and (. .) reference blocks, and all similar blocks, is 11.5 μin/in/°C, a "handbook" value. A temperature change of 5 °C is a change of 1150 μ in for a 20 in block. The present uncertainty of the predicted values at 20 °C is approximately 3μ in. (See table 31 in section 6.5.) In order to demonstrate closure at 25 °C, the error in the differential coefficient of expansion must be substantially less than 3μ in. It may well be necessary, if such closure is required, to determine by separate experiment the coefficient of expansion for each block.

[15] A similar experience is described by J. C. Moody in reference [18].

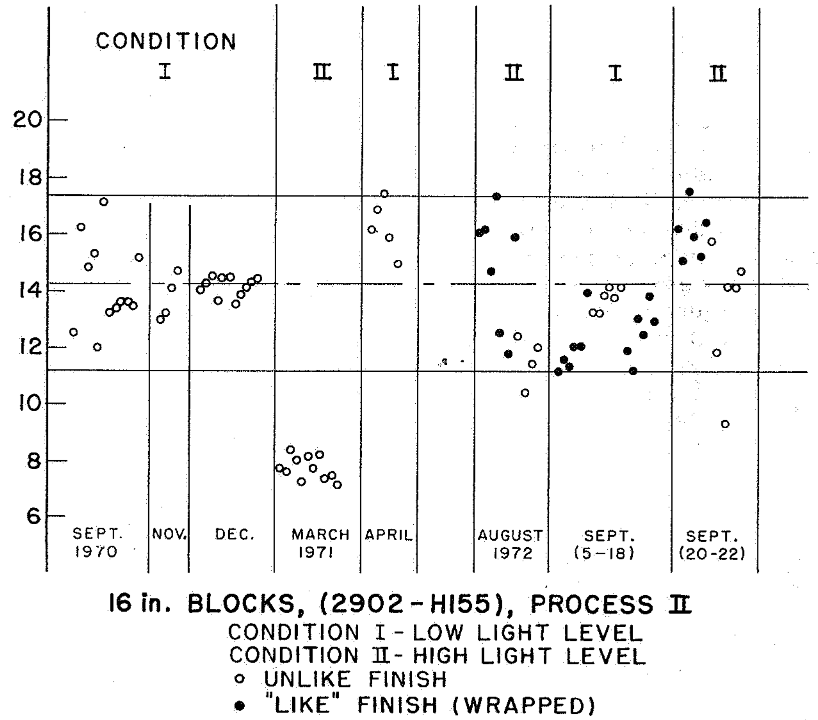


FIGURE 12. *Values for 16 in block determined in laboratories.*

surface properties. That is, blocks which responded differently in the unwrapped condition would respond very nearly alike in the wrapped condition when subject to the same "thermal shock" [16]. The procedure of routinely wrapping long blocks was adopted.

Some time later, it was decided to repeat the initial experiment. The same 16 in blocks were compared in both the "wrapped" and "unwrapped" condition in the same two laboratories. The results are shown in figure 12 over the dates 8/23/72 to 9/22/72. The dashed line shown is the approximate average of the "unwrapped" values obtained at NBS over the period September to December 1970. The limits shown are based on current process performance parameters (January 1974). The March 1971 "unwrapped" values under condition II are clearly outside of these limits. Three 1972 values are "borderline", two "wrapped" and one "unwrapped." One "unwrapped" value is clearly out of these limits. While these measurements do not show a significant difference between the "wrapped" and "unwrapped" results in the well lighted laboratory, all long blocks are measured in the "wrapped" condition at NBS.

A sequence of measurements was made on a group of 8 in blocks, including the 8(.) and the 8(. .), in which an attempt was made to monitor the change in temperature of each block in the course of the series of comparisons. Differential temperatures were measured with thermocouples located in the holes of the "hoke" type blocks. Plastic plugs were used to reduce any "chimney" effect. The data was reduced in two ways: first by assuming all of the blocks to be at the ambient temperature of the laboratory, and then by normalizing the data to a fixed temperature using the differential temperatures and the assumed coefficients of expansion. A statistical analysis of the result, summarized in table 8, does not indicate any significant difference between the two methods. The scheduled sequence of measurements required by the comparison design can be completed in a matter of minutes. Each block is handled about the same amount of time. For these reasons, the actual change in temperature is small, and further, if all blocks are of the same

TABLE 8. *The comparison of results between assuming the temperatures of the blocks to be ambient, and adjusting the data by means of differential temperature measurements to the temperature of the first block.*

	Uncorrected data	Differential temperature correction made
Number of measurements	112	98
Average diff. (8(.)–8(. .))	44.24	44.29
S.D. of average	.072	.073
S.D. of process	.763	.726
Range	3.45	3.60

material as is the usual case, and the temperature change for each block is about the same, the results are not affected. Inasmuch as monitoring the temperature of each block is a difficult procedure, the practice has been discontinued.

The present procedure is to place the wrapped long blocks in the comparator for a period of time to allow thermal equilibration. This is usually done the evening before the scheduled measurements. To evaluate the effect of time intervals between measurement series on a given group of blocks of the same nominal size, a sequence of five series of intercomparisons have been made on a given day with a four-hour interval between the first and second, a two-hour interval between the second and third, a one-hour interval between the third and fourth, and a half-hour interval between the fourth and fifth. This has been done many times, using blocks of all sizes. There is no obvious correlation between the results and the elapsed time interval. Only four blocks of the same nominal size can be stored in the present comparator. The success of these studies, however, indicates that, with adequate storage for 32 blocks, the series of comparisons for each size could follow in sequence with nominal delay.

5.6. "Practical" and "Virtual" Surfaces

Two types of surfaces are of interest in all measurements involving gage blocks, "practical" surfaces, and "virtual" surfaces. "Practical" surfaces establish the position of the gage block relative to a mating object. The "virtual" surface relates to the method of surface detection. The "practical" surface is established by the geometry of the mating surfaces and the procedural steps used in bringing the objects together. Each method of surface detection relies on a different reaction in the interface between the object surface and the detector, and as a consequence each method has its own "virtual" surface. For these reasons, the "virtual" surface and the "practical" surface can never be in coincidence. For a given block, the degree to which the separation between the two is a factor in the measurement depends upon the precision of the particular measurement process.

In both Process I and Process II measurements, the position of the block relative to the platen, or anvil, is established by the "practical" surface located between the bottom of the block and the mating object. In Process I, the "virtual" surface of the gaging face is established by the mechanism which causes a light beam to be reflected from a relatively smooth, contaminated metallic or nonmetallic surface. In the Process II measurements, the "virtual" surface is located in the interface between the surface of the contacting probe and the deformed surface of the gaging face, the deformation occurring because of the force acting on the probe.

The nature of the gage block surface in a normal environment is quite complex. From the outside and progressing inward, with perhaps no one layer being completely continuous, one would expect to find a layer of more or less tightly bound dust particles, residue from some intentional "cleaning" procedure, a layer of fluid or semifluid material consisting of residue from polishing or other surface treatments and previous environments in which the block has been, and interface of cleavage planes, attached pieces of shattered grains, oxides, nitrides and the like and finally the basic material of the block. The detailed geometric features of a large area of such a surface establish in part the "practical" surface. A quantitative description of such a surface must, at the present, be inferred from nondestructive tests over small portions of the area. In essence, there are only two practical tests; the ability to wring to other blocks and platens, and the ability to observe interference fringes.

The nature of how two surfaces come into contact can be illustrated by first considering how a planar surface would come into contact with a surface such as illustrated in figure 13. If the area of the planar surface is small, the "practical" surface would be established by the highest elevation points, a, b, and c. For a larger planar surface, contact would be made at a', b' and c', and for a still larger area, the contact is at a'', b'' and c''. Clearly, only the maximum elevation peaks on the rough surface are involved. The depths of the valleys are immaterial. In such a situation, the "practical" surface is established by the three highest peaks on the rough object which lie within the projected area of the mating planar surface.

Some insight as to the location of the "practical" surface between two mating surfaces can be obtained in the following manner.[16] Let a surface be represented by a grid in which the deviation of each element of the grid above some reference plane is $a_{ij}=C+N_{ij}(\mu,\ \sigma^2)$ where $N_{ij}(\mu,\ \sigma^2)$ is selected at random from a table of normal deviates with average, μ, and variance σ^2. With two such surfaces, arranged so that the reference planes are parallel and separated by an amount y_o (no contact), one can reduce y until contact is obtained. A pair of mating rows from the grids representing two surfaces is shown in figure 14. At this point, the separation between the summation $a_{ij}+b_{kl}$ (where b_{kl} represents the profile of the mating surface) is a maximum value for the initial, or "first", contact between the two surfaces. A future reduction in y will identify the "second" contact, the "third" contact, and so on.[17]

[16] Siddall and Willey, reference [17], discuss a different approach in which surface traces are matched.

[17] For simplicity, it is assumed that the two elements which make initial contact "compress."

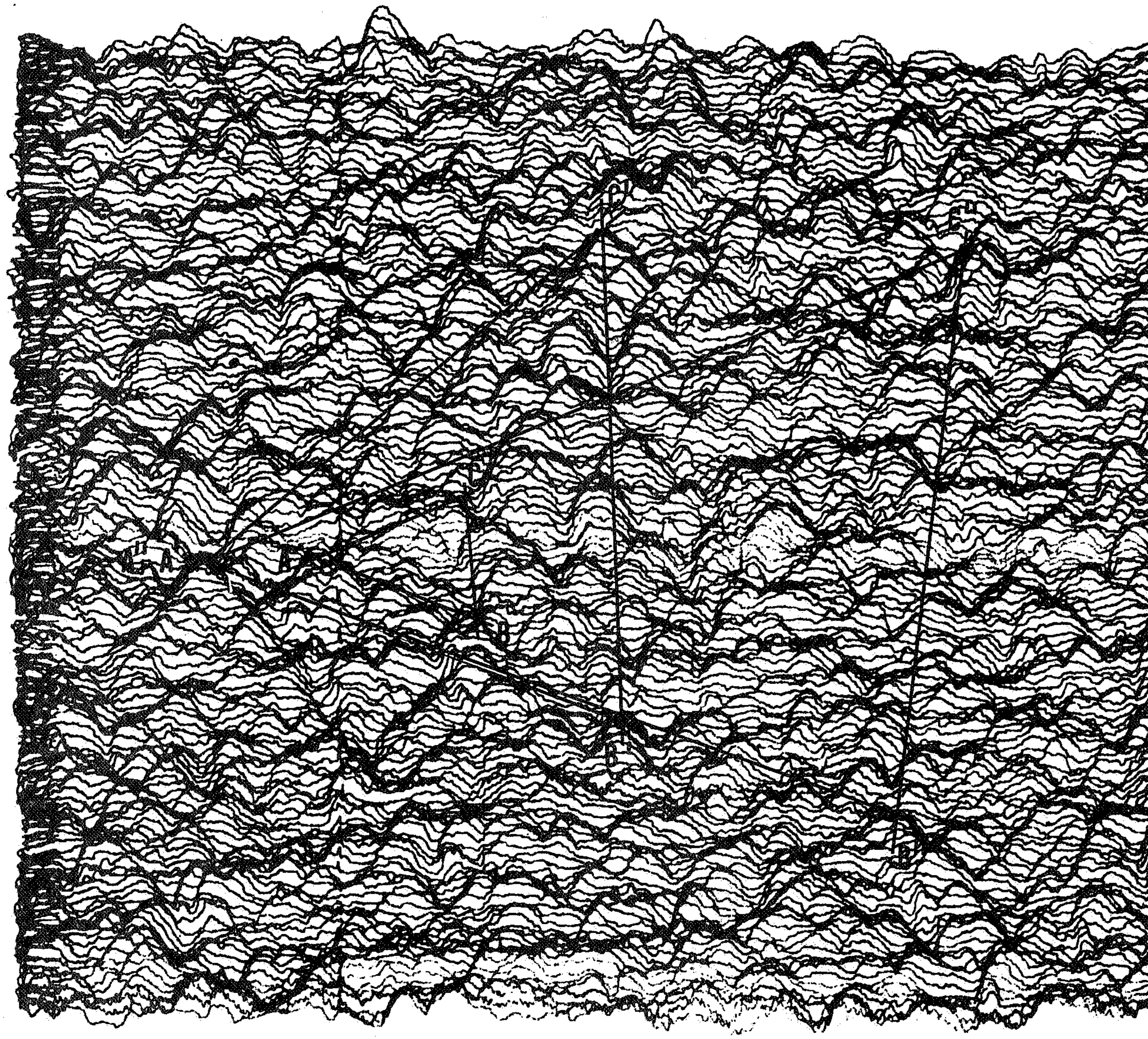


FIGURE 13. *Contact between "random" surface and ideal plane. (Courtesy of Gould Measurement Systems).*

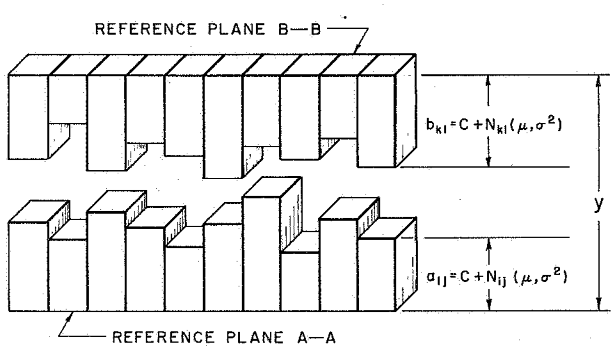


FIGURE 14. *Model surface contact.*

In the above simulation, for $C = 10$, $N_{ij}(0, 1)$, and $N_{kl}(0, 1)$ the maximum expected separation at initial contact, $(a_{ij} + b_{kl})$, would be of the order of $(10 + 3\sigma + 10 + 3\sigma) = 26$. Table 9 summarizes the results for 100 pairs of 10×10 grids, and 20 pairs of 30×30 grids. The table indicates the value of y for "first", "second" and "third" contacts. The degree of "interpenetration" and possible "tilt" is reflected in the results. As long as the surface areas are large, the degree of penetration will be a function of the elevation of the peaks above some reference plane for the "smoothest" block. This infers a stable, reproducible "practical" surface in the interface between the bottom of the block and the comparator anvil provided that both

Table 9

Maximum Separation Between Random Surfaces (Model)

For 10 X 10 Grid, 100 Pairs (20 000 Random Numbers)

	"First" Contact	"Second" Contact	"Third" Contact
Maximum	25.2	24.0	23.5
Minumum	22.6	22.2	21.8
Average	23.5	23.3	22.5

For 30 X 30 Grid, 20 Pairs (36 000 Random Numbers)

	"First" Contact	"Second" Contact	"Third" Contact
Maximum	25.5	24.8	24.4
Minumum	23.6	23.6	23.5
Average	24.5	24.2	24.0

surfaces are clean and free of "burrs." (The blocks and the comparator anvil are cleaned and "stoned" lightly prior to all Process II measurements).

The nature of the "practical" surface in the interface between the block surface and mating platen is altered by the presence of a "wringing" fluid. For a given measurement, the film thickness in the interface, whatever it might be, is included in the initial assignment of a length value by an interferometric process. The variability of a collection of repeated measurements reflects in part the variability of this film thickness. The development of micro-scratches in the surface of the block by virtue of the sliding action necessary to make the "wring" indicates that, at least part of the time, there is an interpenetration of the two surfaces similar to the previous argument. Eventually, surfaces deteriorate to the point that they will no longer "wring." While there is a possibility that some of the damage may occur because of the abrasive action of foreign material on the surfaces, this suggests that for minimum or "zero" film thickness, the "practical" surface between the block and the platen is essentially the same as the practical surface between the block and the comparator anvil.

The maximum "film thickness" is largely a matter of operator "feel" at the time of making the "wring." As a consequence of this added variability, the position of the gage block on the comparator anvil may well be more reproducible than its position as "wrung" on a platen. The standard deviations of the two processes tend to support this conclusion. ("Wringing film thickness" is discussed further in section 7.4.) The practice, after cleaning and "stoning" the long blocks, is to "wring" to a quartz flat and judge the quality of the "wring" by its appearance as viewed through the flat. If all is in order, the quartz flat is removed and the block immediately "wrung" to the appropriate platen.

Because of the operations necessary to obtain a highly reflective surface, figure 15 may be more representative of the block surface profile. In Process I measurements, light waves are reflected from such a surface. Typical gage block interferograms are shown in figure 16 [18]. The presence of surface scratches is evident in most of the interferograms, but the surface from which the light appears to come is not in coincidence with the "practical" surface of the gaging face.

In the case of reflected light, the location of the "virtual" surface is thought to be a function of the roughness of the surface and reaction of the light with the surface molecules. In the first case, interference occurs over a large area so that, with the exception of the edges of the fringe, the detailed surface profile is not revealed. It is sometimes assumed that the reflection plane is located about midway between the peaks and the valleys. In the second case, in the process of absorbing and reradiating the incident light beams, the phase relation between the incident and reflected ray may be changed. The net result of the two effects, which are inseparable, is a "virtual" reflecting surface which cannot be in coincidence with the "practical" surface.

In the Process I measurements, the "reflecting virtual" surfaces are located at both the gaging face and the platen face. As long as the separation between the "virtual" surface and the "practical" surface on both of these faces is nearly the same, the separation between the two "virtual" surfaces is essentially the same as the separation between the two "practical" surfaces. Defining $S(g)$ as the separation between the "virtual" surface and the "practical" surface of the gaging face, and $S(p)$ as a like separation at the platen face, one is concerned as to the significance of $[S(g)-S(p)]$ relative to the precision of the measurement process, for various combinations of blocks and platens. Early studies on short gage blocks under 4 in, reported in reference [19], utilized the "slave block" technique. Later studies used short blocks of various manufacture and two steel platens with different surface finishes. In both cases there was no evidence to indicate that $S(g) \neq S(p)$. In the Process I measurements of long gage blocks, the platens used are made from the same type of material and have the same surface finish as the blocks. It is assumed that $S(g) = S(p)$. (This is not the case when the results from a steel platen are compared with the results from a quartz platen [20].)

The "virtual" surface in the Process II measurements is in the interface between the surface of the contact probe and the deformed gaging surface of the block, as shown in appendix 4. Defining the separation between this surface and the "practical" surface of the block as penetration, one is concerned with the difference in penetration, β, from block

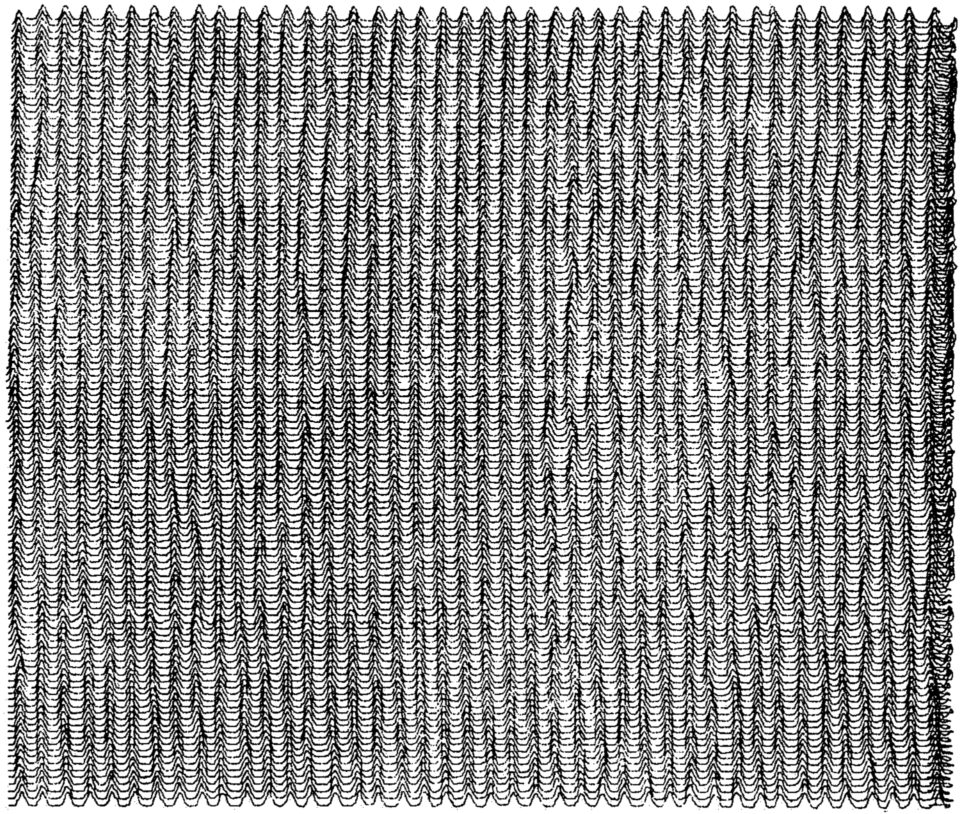


FIGURE 15. *"Typical" gage block profile. (Courtesy of Gould Measurement Systems).*

to block. Factors which determine the magnitude of the penetration are the geometry and physical characteristics of the contacting probe, the geometry and physical characteristics of the block surface, and the contact force. In the transfer of the length of one block to another, as long as both blocks respond in a similar manner to a fixed force on a given probe, β is essentially zero, and the difference in separation between the "virtual" surfaces of the blocks and the reference plane of the comparator is very nearly the same as the difference in separation between the "practical" surfaces of the blocks and the reference plane.

Commercially available long gage blocks are made from through-hardening steel, such as Type W–1 tool steel or Type 52100 steel. Blocks made from such steels, when properly heat-treated, are sufficiently hard for resistance to wear, can be polished to obtain a suitable surface finish, and exhibit a high degree of stability with time [21]. The physical properties of these materials are very nearly the same. One would expect the penetration of a given comparator probe on any pair of steel blocks to be about the same so that β would be very nearly zero. The closure studies between the Process I and Process II measurements, discussed in section 6, verify that β is not large relative to the precision of the process. It is assumed that $\beta = 0$, therefore the small variation in penetration across the surface of the block, and from block to block, is a component of the process variability. This assumption is not true when transferring the value from a steel block to a block made from grossly different material. Gage blocks of nominal length 4 in, and under are commercially available in cervit, chrome-carbide, tungsten-carbide, as well as steel, therefore a detailed discussion of β is included in reference [20].

6. Developing a Measurement Process

6.1. Restraint Requirements

The comparison designs discussed in section 5.3 require two "known" blocks in each sequence of comparisons. The length values assigned to these "known" blocks introduce the measurement unit into the process. That is, the sum of the values for the "knowns" is the restraint for the least squares solution used to determine values for each of the four blocks in the particular measurement sequence.

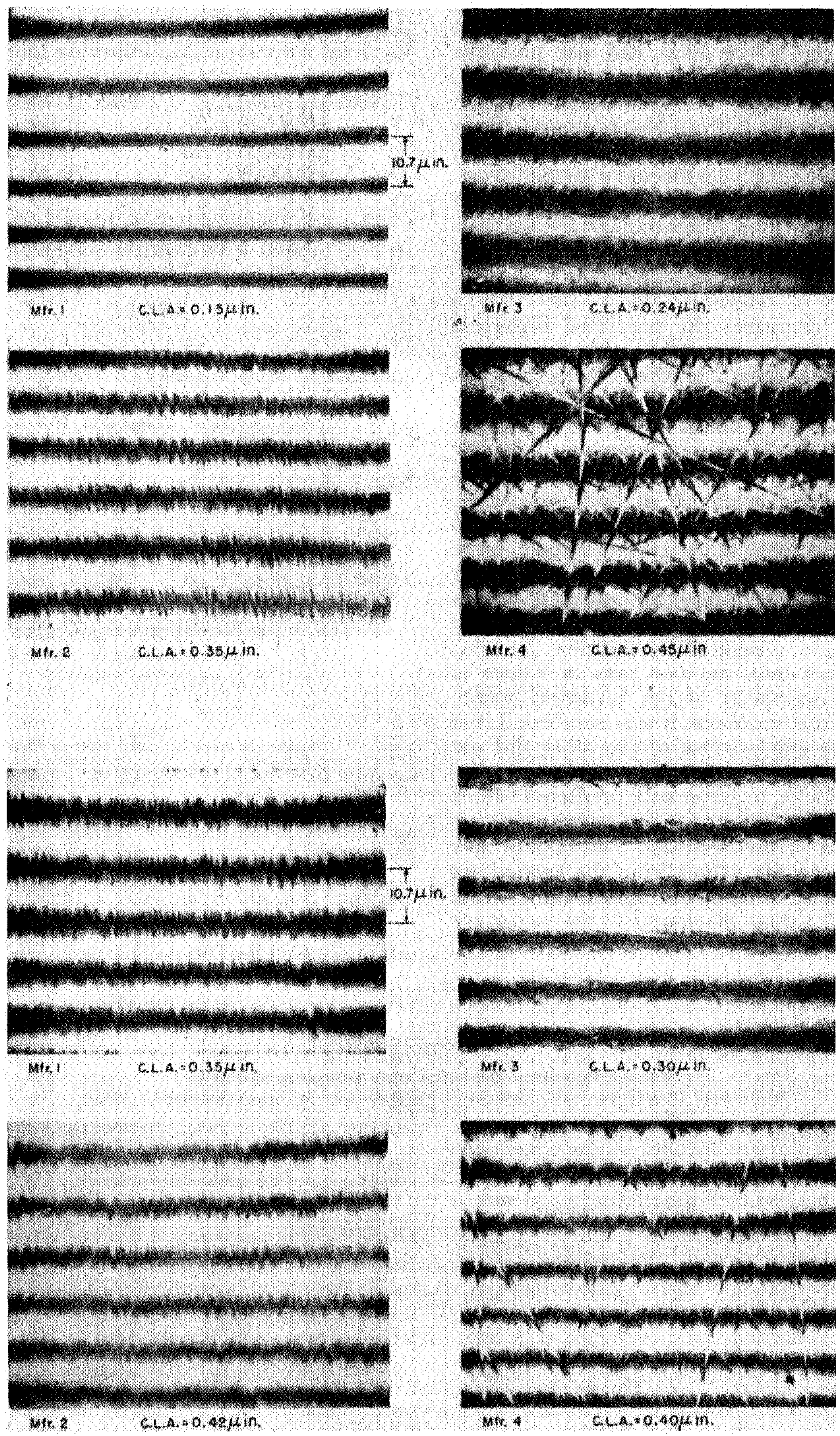


FIGURE 16. *Gage block surface interferograms.*

Using two "known" blocks to introduce the unit rather than one provides a means for monitoring one with reference to the other and also provides a collection of "repeated" measurements which, in turn, reflects the long term performance of the process. Accepting the NBS (.) blocks as one group of "knowns," the task is to establish suitable values for a second set (the NBS (..) blocks) with the "new" interferometric process. In order to do this, the continuity of the results from the historical interferometric measurements and the "new" interferometric process must be demonstrated.

For one group of reference blocks, the NBS (.) group, table 10 compares the predicted historical values discussed in section 4.3 with the values established by the "new" interferometric process (Process I) discussed in section 5.2. With one exception, the area of doubt associated with the historical predicted value encompasses the new process value. For the 8 (.), the uncertainty bands overlap. Within the precision of both processes, continuity appears to be preserved. As an additional check on the continuity of the two processes, table 11 compares the historical values established for the USN blocks with values for selected blocks established by the "new" process. Again, for the blocks which were measured by the new process, the difference between the two sets of values is less than the uncertainty of the historical value. On the basis of this evidence, it was concluded that the change from one process to the other did not affect the continuity of the measurements.

The NBS (.) blocks, together with predicted values based on "new" process measurements, are accepted as part of the restraint. The task is now to establish acceptable values from the second set of blocks necessary to complete the restraint. From this point on, all values discussed in the paper are from the "new" process.

The NBS(. .) group of blocks was selected for the second required set of reference blocks. The (. .) set consists of the following blocks:

NBS H178-5(. .)	NBS H148-10(. .)
H132-6(. .)	H249-12(. .)
H105-7(. .)	H155-16(. .)
H143-8(. .)	H146-20(. .)

These are relatively new blocks which were used in conjunction with multiple wavelength interferom-

TABLE 10

Summary Comparison (.) Values, "Old" (Y_H) vs "New" (Y_I)

Block Ident.	(a) 7/01/71 Predicted Value Based on Historical Data	(b) 1/01/72 Average Value from New Process	Diff. (a)-(b)	UNC* 7/01/71	UNC** 1/01/72
NBS(.)					
M136-5	29.5	29.3	.2	1.2	1.4
M115A-6	28.3	29.4	-1.1	1.6	1.4
M202A-7	14.9	15.5	- .6	1.9	1.9
M103A-8	47.9	50.7	-2.8	2.2	1.9
M109A-10	56.0	58.4	-2.4	2.8	1.9
M135A-12	70.8	71.9	-1.1	3.5	1.9
M109A-16	66.8	67.7	+ .1	4.8	1.9
A157-20	1.6	1.1	+ .5	6.1	1.9

* OF PREDICTED VALUE (SEE TABLE 4)

** OF AVERAGE (SEE TABLE 7

TABLE 11

Summary of Comparison USN, "Old" vs "New" Values

Block Serial No.	Block Nominal Size n	Y_H(7/1/71,20)	UNC	Y_I(1/1/72,20)	UNC
R317A-5	5 in	19.5	0.8	19.7	1.3
U157A-6	6	14.1	1.1	14.3	1.3
T229A-7	7	43.7	1.2	—	-
W186A-8	8	18.5	1.4	19.6	1.3
V215A-10	10	14.9	1.8	—	-
U136A-12	12	38.7	2.2	—	-
W234A-16	16	15.5	3.0	—	-
W198A-20	20	37.0	3.8	39.1	1.1

Y_H (SEE TABLE 2)

TABLE 12

NBS(..) Reference Standards with Reference to NBS(.)
(Mechanical Comparison with Restraint for Solution on Value for NBS(.) Only)

Nominal Size	NBS(.) Y_I Values* (Restraint)	NBS(..) Y_{II} Values** 11/71 Series	12/71 Series	Avg.	NBS(..)Y_I Values*	Diff.
5	29.3	1.1	- .2	.45	.9	- .45
6	29.4	-2.9	-1.3	-2.1	-2.1	0
7	15.5	-5.8	-4.3	-5.0	-5.8	+ .8
8	50.7	6.8	8.1	7.4	7.4	0
10	58.4	-1.8	-2.0	-1.9	-1.4	- .5
12	71.9	14.2	14.9	14.6	14.9	- .3
			13.4			
16	67.1	10.2	10.7	10.4	10.3	.1
20	1.1	18.9	16.5	17.6	21.7	-4.1
		17.7	17.2			

* See Table 7

** 4 degrees of freedom per series

TABLE 13

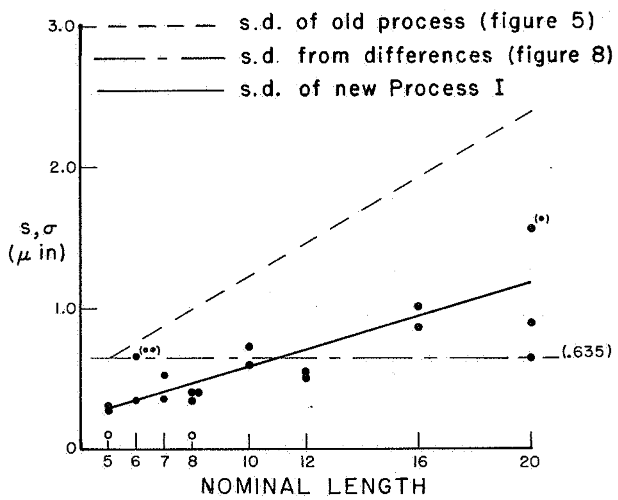
NBS(.) and (..) Accepted Values, Sums and Differences, November 22, 1972

Nominal Size	Y_I Values-Nov. 22, 1972 (.) Group Ave	n	Δ	(..) Group Ave	n	Δ	Restraint Y_{II} (.)+(..)	"Check" Y_{II} (.)-(..)	UNC*
5	29.0	5	-.3	1.1	6	+.2	30.1	27.9	.54
6	29.1	4	-.3	-2.4	4	-.3	26.7	31.5	.72
7	15.6	3	.1	-5.7	4	.1	9.9	21.3	.90
8	50.6	3	-.1	6.5	4	-.9	57.1	43.6	1.02
10	58.4	3	0	-2.0	4	-.6	56.4	60.4	1.38
12	72.3	3	.4	15.0	4	.1	87.3	57.3	1.50
16	67.3	4	-.4	9.9	4	-.4	77.2	57.4	2.01
20	2.9	6	1.8	21.0	6	-.7	23.9	-18.1	2.31

n Number of independent values in average.

Δ Change from Table 12.

* Uncertainty for both ((.)+(..)) and ((.)-(..)).



$$s = \sqrt{\frac{\Sigma(\text{dev})^2}{N-1}}$$

PROCESS I STANDARD DEVIATION (NOV. 1972)

FIGURE 17. *Process I standard deviation.*

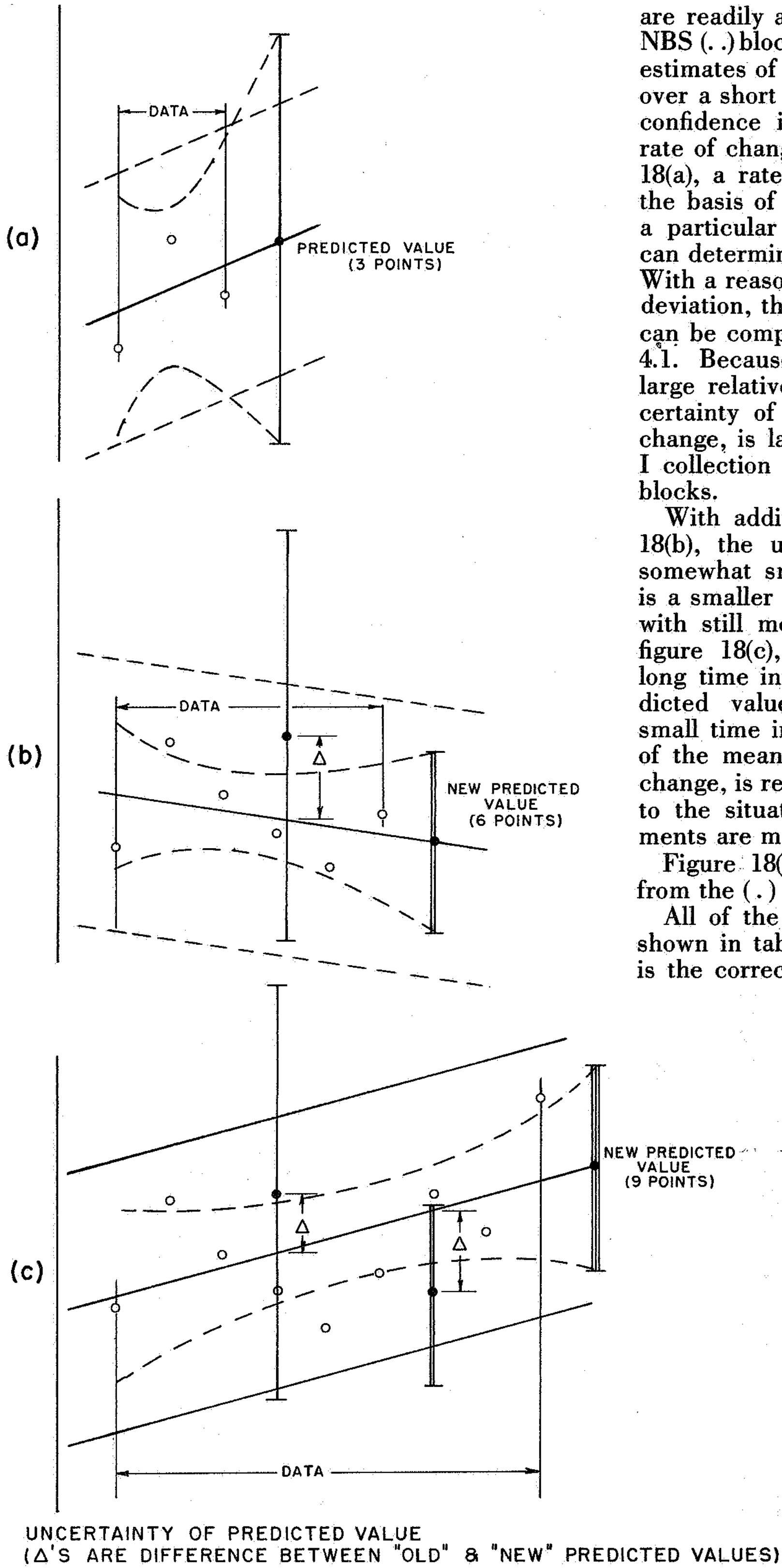
etry but which had no previous history of values assigned by the "old" process. By virtue of the closure between the two processes discussed above, the average values from the "new" process (see table 7) were accepted as the tentative values for the NBS (. .) blocks. Intercomparison measurements with reference to the NBS (.) blocks, summarized in table 12, were made to verify closure between the transfer values relative to the NBS (.) assigned values, and the tentative interferometric values. With the exception of the 20 in blocks, the agreement is remarkably close. The discrepancy at the 20 in level is considered acceptable in view of the small number of measurements available for the assignment of values to both the 20(.) and the 20(. .) blocks.

With additional measurement data available, the accepted values for the individual blocks, the sums and difference for the pairs, and the process uncertainty in use in September 1972, are shown in table 13. The estimated uncertainty tabulated in table 13 is $3(\sqrt{2})\sigma$, where σ is from the "fitted" line on figure 17. The points plotted in figure 17 are the computed standard deviation of the collections of values for each of the (.) and (. .) blocks and the appropriate USN blocks. The dash-dot line, $\sigma = 0.635$, is the original estimate established in figure 8. The dashed line is the estimated process standard deviation for the "old" process established in figure 5.

6.2. Predicted Values (Process I)

Partly as a practical expedient, and partly because it was thought that the relatively small rates of change would not be apparent over the short time span associated with "new" process measurements, changes with time have not been considered up to now. Under the assumption that the length of all of the blocks change with time, the average value is not the best estimate of current or future values. It is necessary to predict appropriate values for individual blocks, sums and differences, together with appropriate uncertainties, over some reasonable time interval. Because closure is an important criteria for judgment, it is necessary to have realistic estimates of uncertainty for the predicted values. This and the following two sections are devoted to establishing and verifying realistic rates of change.

In the case of the NBS (.) blocks, with a long history of measurement, significant rates of change



are readily apparent in the historical data. For the NBS (. .) blocks, with no long history of measurement, estimates of rates of change must be based on data over a short time span. As the data base increases, confidence in both the predicted values and the rate of change will increase. To illustrate, in figure 18(a), a rate of change, or slope, is computed on the basis of three hypothetical measurements over a particular time span. From the fitted line, one can determine a predicted value for any given time. With a reasonable estimate of the process standard deviation, the uncertainty of the "predicted value" can be computed from the formula used in section 4.1. Because the extrapolation time interval is large relative to the "data" time interval, the uncertainty of the predicted value, and the rate of change, is large. This is analogous to the Process I collection of data for the (.) blocks and the (. .) blocks.

With additional data points, as shown in figure 18(b), the uncertainty of the predicted value is somewhat smaller since the extrapolation interval is a smaller function of the new data base. Finally, with still more data available, such as shown in figure 18(c), the data bank covers a sufficiently long time interval that the uncertainty of the predicted value extrapolated over some relatively small time increment approaches the "uncertainty of the mean." At this point, the slope, or rate of change, is reasonably well known. This is analogous to the situation as additional Process I measurements are made.

Figure 18(c) is analogous to the historical data from the (.) blocks.

All of the values available up to June 1972 are shown in table 14. For each block, the value $Y(\mathrm{I})$ is the correction to nominal length as obtained by

FIGURE 18. *(a) Predicted value, 3 data points, (b) predicted value, 6 data points, and (c) predicted value, 9 data points.*

TABLE 14. *Summary of predicted value computations.*

SER NO	NOMINAL	DATE	RES	PRED	3SIG PRED	X(I)*	Y(I)
M136	5.00000000	8 24 71	-.01	29.31	1.49837242	13.6411	29.30
		10 18 71	.02	29.18	1.10141098	13.7890	29.20
		3 6 72	-.04	28.84	1.05066556	14.1808	28.80
		3 7 72	.27	28.83	1.05666725	14.1836	29.10
		3 8 72	-.23	28.83	1.06272294	14.1863	28.60
H178	5.00000000	9 13 71	.15	1.05	1.17258103	13.6932	1.20
		9 30 71	-.27	1.07	1.05748172	13.7397	.80
		10 15 71	.12	1.08	.96562185	13.7808	1.20
		3 6 72	-.40	1.20	1.05533223	14.1808	.80
		3 7 72	.10	1.20	1.06179731	14.1836	1.30
		3 8 72	.30	1.20	1.06830278	14.1863	1.50
M115A	6.00000000	8 25 71	.03	29.37	1.49013871	13.6438	29.40
		10 18 71	-.04	29.24	1.12024894	13.7890	29.20
		3 15 72	-.30	28.90	1.28731512	14.2055	28.60
		3 16 72	.31	28.89	1.29437192	14.2082	29.20
H312	6.00000000	9 14 71	.45	-2.05	1.35908125	13.6959	-1.60
		9 29 71	-.49	-2.11	1.25014015	13.7370	-2.60
		3 15 72	-.38	-2.82	1.29983598	14.2055	-3.20
		3 16 72	.42	-2.82	1.30710523	14.2082	-2.40
W202A	7.00000000	8 26 71	.00	15.50	1.84499115	13.6466	15.50
		4 12 72	-.35	15.65	1.30179745	14.2795	15.30
		4 13 72	.35	15.65	1.30743296	14.2822	16.00
H105	7.00000000	9 15 71	.59	-5.79	1.34569207	13.6986	-5.20
		9 28 71	-.62	-5.78	1.26352671	13.7342	-6.40
		4 12 72	-.18	-5.62	1.30080192	14.2795	-5.80
		4 13 72	.22	-5.62	1.30712561	14.2822	-5.40
M103A	8.00000000	9 3 71	.00	50.60	1.84498999	13.6658	50.60
		4 5 72	-.40	50.60	1.30161662	14.2603	50.20
		4 6 72	.40	50.60	1.30761462	14.2630	51.00
H143	8.00000000	9 10 71	.87	7.43	1.35931462	13.6849	8.30
		9 27 71	-.95	7.35	1.24990916	13.7315	6.40
		4 5 72	.04	6.46	1.30024020	14.2603	6.50
		4 6 72	.04	6.46	1.30668136	14.2630	6.50
M109A	10.00000000	9 1 71	.01	58.39	1.84492531	13.6603	58.40
		4 21 72	-.60	58.40	1.29636528	14.3041	57.80
		4 24 72	.60	58.40	1.31291182	14.3123	59.00
H148	10.00000000	9 9 71	-.09	-1.31	1.35483630	13.6822	-1.40
		9 26 71	.10	-1.40	1.25433275	13.7288	-1.30
		4 21 72	-.21	-2.59	1.29476681	14.3041	-2.80
		4 24 72	.20	-2.60	1.31251545	14.3123	-2.40
M135A	12.00000000	8 31 71	-.00	71.80	1.84499304	13.6603	71.80
		5 17 72	.30	72.50	1.30212003	14.3753	72.80
		5 18 72	-.30	72.50	1.30710903	14.3781	72.20
H249	12.00000000	9 8 71	-.63	14.93	1.34686165	13.6795	14.30
		9 24 71	.67	14.93	1.26235954	13.7233	15.60
		5 17 72	-.22	15.02	1.30128424	14.3753	14.80
		5 18 72	.18	15.02	1.30656852	14.3781	15.20
M109A	16.00000000	8 27 71	.06	67.74	1.30717546	13.6493	67.80
		8 28 71	-.04	67.74	1.30191566	13.6521	67.70
		4 28 72	-1.12	66.92	1.29146101	14.3233	65.80
		5 3 72	1.10	66.90	1.31775805	14.3370	68.00
H155	16.00000000	9 7 71	-.11	10.31	1.34679803	13.6767	10.20
		9 22 71	.14	10.26	1.26227620	13.7178	10.40
		4 28 72	-.97	9.57	1.28983255	14.3233	8.60
		5 3 72	.94	9.56	1.31801963	14.3370	10.50
A157	20.00000000	8 30 71	-.02	1.42	1.84412411	13.6575	1.40
		3 12 72	.65	3.95	1.04944718	14.1973	4.60
		3 13 72	-.17	3.97	1.05456385	14.2000	3.80
		3 20 72	-.46	4.06	1.09261698	14.2192	3.60
H146	20.00000000	9 2 71	-.54	21.74	1.37626223	13.6630	21.20
		9 23 71	.61	21.69	1.23092210	13.7205	22.30
		3 12 72	.10	21.30	1.04716051	14.1973	21.40
		3 13 72	-.19	21.29	1.05269817	14.2000	21.10
		3 20 72	.02	21.28	1.09273252	14.2192	21.30

*Time in years measured from 1958.

TABLE 15. *Process I pooled standard deviation and block rate of change.* S.D. = standard deviation of individual measurement; XBAR = average time; YBAR = average of all values; A = intercept; B = slope; SDB = standard deviation of slope.

SER NO	NOMINAL	S.D.	N	XBAR	YBAR	A	B	SDB	T=B/SDB	AVG S.D.
M136	5.00000000	.61500000	5	13.99616408	28.99999952	41.35402060	-.88267193	1.17416480	-.75174449	
H178	5.00000000	.61500000	6	13.96073031	1.13333333	-3.04497236	.29928991	1.11950527	.26734122	
M115A	6.00000000	.61500000	4	13.96164370	29.09999990	40.77057743	-.83590285	1.22742754	-.68102011	
H312	6.00000000	.61500000	4	13.96164370	-2.44999999	18.53376698	-1.50295821	1.25184469	-1.20059478	
W202A	7.00000000	.61500000	3	14.06940627	15.59999990	12.22211611	.24008717	1.18757106	.20216657	
H105	7.00000000	.61500000	4	13.99863005	-5.69999993	-9.73748553	.28842006	1.08859476	.26494712	
M103A	8.00000000	.61500000	3	14.06301355	50.59999943	50.53418207	.00468020	1.26401117	.00370266	
H143	8.00000000	.61500000	4	13.98493135	6.92499995	30.34133935	-1.67439789	1.10929313	-1.50942779	
M109A	10.00000000	.61500000	3	14.09223723	58.39999962	58.15033579	.01771643	1.16240174	.01524123	
H148	10.00000000	.61500000	4	14.00684917	-1.97499999	26.83147764	-2.05659941	1.01877393	-2.01870045	
M135A	12.00000000	.61500000	3	14.13789940	72.26666641	58.48655987	.97469271	1.05133116	.92710341	
H249	12.00000000	.61500000	4	14.03904092	14.97499990	13.04558635	.13743201	.90968765	.15107604	
M109A	16.00000000	.61500000	4	13.99041080	67.32499981	84.37028885	-1.21835519	.90504555	-1.34618106	
H155	16.00000000	.61500000	4	14.01369846	9.92499995	25.89550614	-1.13963535	.97061710	-1.17413484	
A157	20.00000000	.61500000	4	14.06849301	3.34999996	-62.73942280	4.69769025	1.29518704	3.62703615**	
H146	20.00000000	.61500000	5	13.99999976	21.45999956	33.14868593	-.83490620	1.08955614	-.76628102	
										.61499999

the defined interferometric process. The column $X(\mathrm{I})$ is a time coordinate for the date of measurement referred to an arbitrary "zero" time. The predicted value for each date is that from a least squares straight line fit of Y as a function of X. The residual is the difference ($Y(\mathrm{I})-$Predicted). The 3 sigma predicted column is computed by the formula given in section 4.1. It should be noted that the 3 sigma predicted value is smallest near the centroid of the time span covered by the data. The process standard deviation used for these computations is $\hat{\sigma}=0.615$ from table 15.

Table 15 shows, for each block, the pooled standard deviation (i.e., computed from all of the residuals in table 14) and the number of measurements which have been made for each block. *XBAR* is the location of the centroid of the points according to time from an arbitrary time "zero". *YBAR* is the average of all of the available points for each block. *A* is the intercept value at an arbitrary time "zero" (1 Jan. 1958). *B* is the computed slope or rate of change in microinches per year. The test, $T=B/SDB$, is computed from B and the standard deviation of B, SDB. Except for A157 (20(.)), $T<3$, thus at the present level of precision and over the time interval of these measurements, the rate of change does not appear to be significant. It is of interest to note that the "pooled" standard deviation, $\hat{\sigma}=0.615$ microinches, is in good agreement with the estimate, $\hat{\sigma}=0.635$ microinches computed from earlier successive differences (see figure 8).

The rate of change data are summarized in table 16. The rate of change for the NBS (.) blocks based on new data can be compared with the historical data from table 5. In order to check on the appropriateness of the rate of change estimates for the (. .) blocks in figures 19(a) and (b), all of the available measurement history is shown with respect to a prediction line drawn through the July 1, 1973 (7/1/73) predicted value. The small open circles, and the small circles with the horizontal lines have the same meaning as before. (See sec. 4.2) The point on the 1972 line, and associated uncertainty limits, are the tentative accepted values from

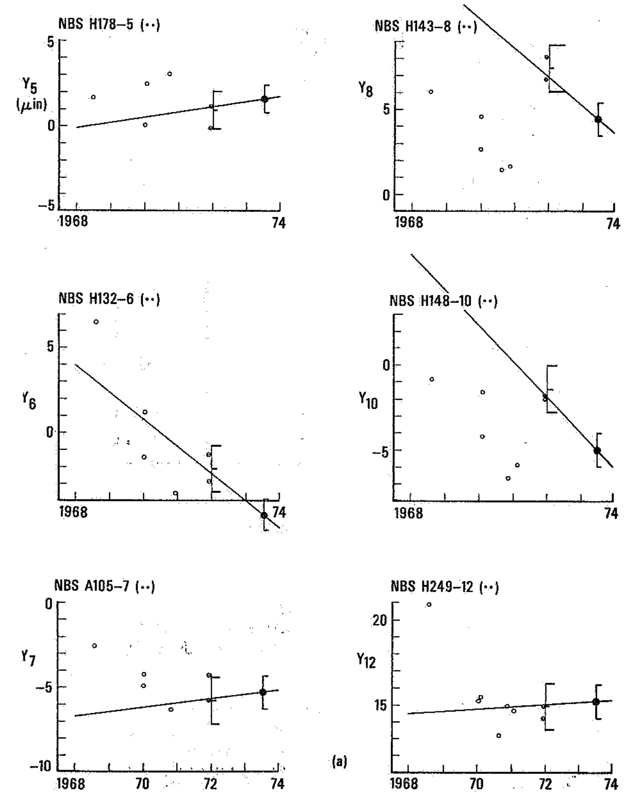


FIGURE 19. *(a) Measurement data, NBS(. .)5, 6, 7, 8, 10 and 12.*

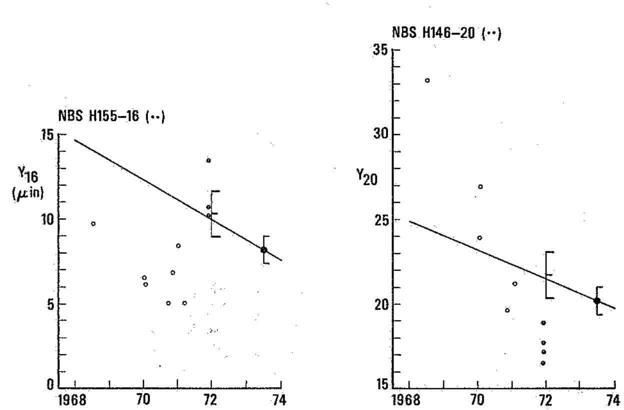


FIGURE 19. *(b) Measurement data, NBS(. .) 16 and 20.*

table 7. For the 7(. .) and the 20(. .) the supporting data appear to differ substantially from the change estimate in table 16. In all other cases, the supporting data seem to substantiate the new estimate, or to be inconclusive.

TABLE 16
Estimated Rate of Change (μin/yr)
(.) and (..) Reference Blocks

Block Ident.	Rate of Change Based on: Historical Data*	New Data
(.)		
136-5	.300	- .883
115A-6	.162	- .836
202A-7	0	.240
103A-8	0	.005
109A-10	.177	.018
135A-12	0	.974
109A-16	.215	-1.218
157-20	-1.478	4.680
(..)		
178-5	-	.300
312-6	-	-1.503
105-7	-	.288
143-8	-	-1.674
148-10	-	-2.057
249-12	-	.137
155-16	-	-1.140
146-20	-	- .835

* From Table 5

TABLE 17
Comparison of Predicted Process I Values with
Average Process I Values Which Have Been Used in Process II

Block Ident.	Accepted Historical Value	Accepted Value 2/23/72	Accepted Value 12/8/72	Predicted Value 7/1/73
(.)				
136-5	29.5	29.3	29.0	29.4*
115A-6	28.3	29.4	29.1	29.4
202A-7	14.9	15.5	15.6	15.6
103A-8	47.9	50.7	50.6	50.6
109A-10	56.0	58.4	58.4	58.6
135A-12	70.8	71.9	72.3	72.3
109A-16	66.8	67.7	67.3	67.6
157A-20	1.6	1.1	2.9	1.5
(..)				
178.5	—	0.9	1.1	1.6**
312-6	—	-2.1	-2.4	-4.8
105-7	—	-5.8	-5.7	-5.3
143-8	—	7.4	7.0	4.4
148-10	—	-1.4	-2.0	-5.0
249-12	—	14.9	15.0	15.2
155-16	—	10.3	9.9	8.2
146-20	—	21.7	21.0	20.2

* Based on historical rate of change.

** Based on Process I rate of change estimates.

Table 17 shows a comparison of the predicted values, accounting for change with time, for 7/1/73 with all previous "accepted," or average, values. Table 18 lists the predicted values for the sums and differences, as required in the comparison process, together with the uncertainty of these values based on the "pooled" standard deviation for the process. These predicted values are monitored in two ways. First, it is expected that the results from future Process I measurements will verify the prediction. For example, referring to the Δ's in figure 18, a value predicted back to the time of the last prediction (based on the smaller data base) is a check on the continuity of the data.

Second, a value predicted forward in time based on the increased data base provides the necessary restraint data for Process II, and the difference measurements from Process II should verify the differences between the appropriate Process I predicted values. Failure in either case is an indication of the existence of a problem. One, or both, blocks may have changed in an unexpected manner, or the predicted values are in error. The values shown in tables 16 and 18 will be revised later in this paper as a result of both additional Process I measurement data, and Process II difference measurements.

TABLE 18

Predicted Sum and Difference Process I Values
for July 1, 1973 as of December 1972

SUM (.)+(..)	DIFF (.)-(..)	UNC*
31.0370	27.8489	1.58
24.5889	34.1131	1.84
10.3330	20.8670	2.00
54.9882	46.2118	2.00
53.5988	63.6904	2.00
87.4424	57.0909	2.00
75.8741	59.4118	1.84
21.6850	-18.7303	1.75

* Applies to both SUM and DIFF

6.3. Process II Performance Parameters

In order to compare results from Process I, such as the computed difference ((.)−(. .)), with Process II results, the measured difference ((.)−(. .)), it is necessary to establish Process II performance parameters. The Process II within-group standard deviation, and total standard deviation, in addition to being used to monitor the process performance determine in part the uncertainty to be associated with the process output. Initially, the magnitude of these parameters is unknown. In practice, one starts with estimates based on short sequences of repeated measurements and then modifies the estimates as a history of process performance develops. If all is going well, the estimates approach long term stable values which become the accepted parameters for the particular process.

The redundancy of the intercomparison design provides the mechanism for establishing the within-group standard deviation which is the standard deviation of a "single" comparison computed as shown in appendix 3 and reference [6]. For the designs used each estimate of the within-group standard deviation is based on 5 degrees of freedom. These estimates, combined for many series of measurements, establish the accepted within-group standard deviation, σ_w, as shown in table 19. Since there is no immediately apparent reason why the accepted standard deviation for the 6 in, 7 in, 10 in, and 12 in blocks should be less than that for the 5 in and 8 in blocks, for control purposes, the accepted standard deviation was "rounded" as shown. In like manner, the accepted standard deviation for control purposes for the 16 in and 20 in blocks was also "rounded."

TABLE 19
Within Group Standard Deviation, Comparative Process

Nominal Size	No. of Series*	Observed S.D.	Accepted S.D. for Control Purposes
5 in	77	.50	.5
6 in	132	.39	.5
7 in	89	.42	.5
8 in	173	.5	.5
10 in	72	.41	.5
12 in	73	.48	.5
16 in	52	.49	.8
20 in	62	.67	.8

* 4 degrees of freedom per series

For each new sequence of measurements, the computed or observed standard deviation is tested for conformity with the existing distribution by computing an "*F* ratio:"

$$F = ((\text{Observed S.D.})/(\text{Accepted S.D.}))^2$$

If the "*F* ratio" exceeds a suitable limit, an "out-of-control" situation for that particular sequence of comparisons is indicated, and the measurements must be repeated. If the accepted standard deviation, σ_w, really reflects the process performance, few measurements will have to be repeated. On the other hand, a few "out-of-control" measurements occurring in sequence following a long sequence of "in-control" measurements are an almost sure sign of process troubles.

The difference, ((.) − (. .)), is determined in every comparison measurement series. For a given pair of reference blocks, this difference should be reasonably well behaved, and similar to all other difference measurements required in the particular design. A collection of measurements of this difference reflects the total variability of the process over the time span of the collection. The standard deviation of such a collection is the total standard deviation, σ_T, of the process. The total standard deviation reflects the difference between the variability accounted for in the measurement algorithm, the within-group standard deviation, and the variability from all sources which affect the process over time. The total standard deviation is a measure of the ability of the process to repeat a given measurement.

As an initial estimate, the accepted Process II difference between the reference blocks was the average of a collection of measured differences. Each series of measurements produces a new value for this difference. The new value is checked for conformity with the existing collection by computing a "*t* ratio:"

$$t = (\text{New Obs. Diff.} - \text{Accepted Diff.})/(\text{Accepted Total S.D.})$$

For *t* values exceeding suitable limits, the process is considered "out of control." On the assumption that the estimates of the accepted difference and the accepted total standard deviation are proper, an "out of control" signal indicates that one or the other of the reference blocks has changed or, for some reason or other, the process is not measuring the appropriate differences.

The accepted total standard deviation as of July 1973 is tabulated in table 20. On the initial assumption that difference is constant, the average measured ((.) −(. .)) is tabulated together with the number of values in each average. These averages are the

TABLE 20
Total, or Process, Standard Deviation, Comparative Process

Block Designation	(.)-(..)* Average	No. of Series	Accepted Total S.D.**	S.D. for Control	S.D. of Mean
M136-5 - H178-5	28.5	61	.46	.5	.059
M115A-6 - H312-6	32.1	111	.47	.5	.045
M202A-7 - H105-7	22.0	58	.34	.5	.045
M103A-8 - H143-8	44.2	112	.76	.8	.072
M109A-10 - H148-10	61.4	66	.65	.8	.081
M135A-12 - H249-12	57.0	62	.91	.8	.115
M109A-16 - H155-16	57.5	40	.79	.8	.125
A157-20 - H146-20	-16.5	54	1.14	1.1	.156

* Tentative pending check on closure between Process (I) and Process (II).

** Degrees of freedom is one less than the number of series.

"Accepted Differences" used in computing the "*t* ratio." The total standard deviation, for control purposes, was "rounded" as indicated.

6.4. Closure

When a particular measurement is made by each of two different measurement processes, it must be shown that the two results "close." If both processes are indeed measuring the same thing, as a minimum, the uncertainty limits associated with each result should overlap. Process II, the comparison process, measures the difference ((.)−(. .))II directly. Process I, the "new" interferometric process, established values for each of the (.) and (. .) blocks from which the difference ((.)−(. .))I can be computed. These two sets of differences are tabulated in table 21. The uncertainty of the difference as shown for ((.)−(. .))I is from table 18. The uncertainty as shown for ((.)−(. .))II is 3 times the standard deviation of the mean, from table 20. For the 6 in blocks, $[((.)-(..))I-((.)-(..))II]=34.1-32.1=2.0>(\mathrm{UncI}+\mathrm{UncII})=(1.84+0.135)$, therefore the results do not "close." The same is true for the 10 in blocks. Closure is marginal for the 8 in, 16 in and 20 in blocks.

TABLE 21

Closure, Process (I) and Process (II)

Nominal Size	Process (I) (.)-(..) (Predicted July 1973)	Process (I) Uncertainty of Diff. (Table 18)	Process (II) Average (.)-(..)	Process (II) 3 S.D. of Mean
5 in	27.8	1.58	28.5	.177
6 in	34.1	1.84	32.1	.135*
7 in	20.9	2.00	22.0	.135
8 in	46.2	2.00	44.2	.216
10 in	63.7	2.00	61.4	.243*
12 in	57.1	2.00	57.0	.345
16 in	59.4	1.84	57.5	.375
20 in	-18.7	1.75	-16.5	.468

* Differences, as established by Process (I) and Process (II), do not agree within expected limits.

In retrospect, for the (.) blocks the rate of change used to establish the predicted values was based on a long history, (table 5), and even if some of the historical points have questionable uncertainty limits, the time span covered (with the exception of the 20 in block) adds confidence in the rate of change. On the other hand, for the (. .) blocks the rate of change used was from the small collection of Process I data. In the case of Process II, the average difference ((.)−(. .)) is based on a large collection of values, the difference being determined in every sequence of measurements. In table 21, the computed difference between the predicted Process I values has been compared with the average Process II difference. While the precision of Process II is smaller than that of Process I, because of the short time span covered by these measurements, initially it did not seem likely that one could detect the rate of change of the differences between two blocks.

The ((.)−(. .)) data from Process II was analyzed to determine the rate of change of the difference, the standard deviation of this rate of change, and a predicted difference for July 1973 (7/73) and January 1974. The results of this analysis are summarized in table 22. In the case of the 5 in, 8 in, 10 in, and 20 in blocks, the rate of change is not significant relative to the standard deviation of the rate of change. The standard deviation of the collection and of the fit are essentially the same. For the rest of the blocks, the rate of change is considered to be significant relative to the S.D. of the rate of change (slope). This is further verified by the fact that the standard deviation about the fitted line is smaller than the standard deviation of the collection.

TABLE 22

Analysis of Process II ((.)-(..)) Data
(Same Data as Used in Table 20)

Nominal Size	Assuming Change With Time: Fitted Slope	S.D. Slope	Accepted Slope	* S.D.	Assuming Constant: ** S.D.
5 in	.038	.195	0	.468	.465
6 in	.907	.178	.907	.472	.522
7 in	1.465	.259	1.465	.592	.679
8 in	.191	.266	0	.803	.802
10 in	- .088	.264	0	.718	.715
12 in	- .815	.330	- .815	1.059	1.087
16 in	-1.588	.388	-1.588	.900	1.000
20 in	.404	.410	0	1.175	1.087

* Of collection about fitted line

** Relative to average value, no fitting

TABLE 23

Summary Rate of Change Computation

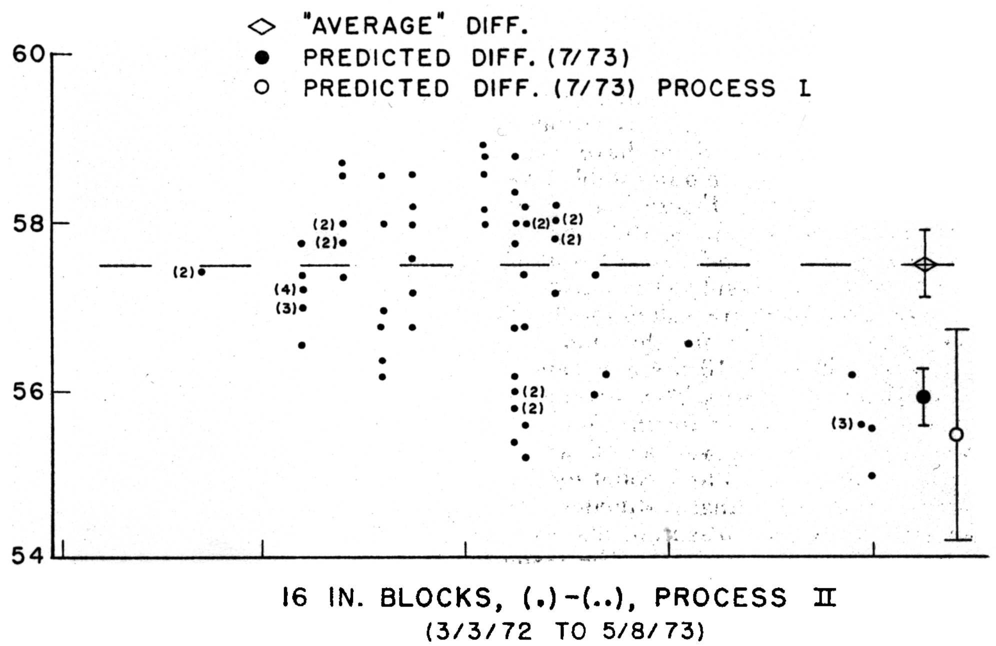
Nominal Size	Hist. (.) (a)	Process I Estimate (.)	Process I Estimate (..) (b)	Process I Computed (.)-(..) ((a)-(b))	Process II (Measured) (.)-(..) (c)	New Estimate (..) ((a)-(c))
5 in	.300	- .883	+ .300	- .005	0	.30
6 in	.162	- .836	-1.503	1.666	.90671	- .74
7 in	0	.240	.288	- .288	1.46455	-1.47
8 in	0	.005	-1.674	+1.674	0	0
10 in	.177	.018	-2.057	+2.231	0	.18
12 in	0	.974	.137	- .137	- .81543	- .82
16 in	.215	-1.218	-1.140	1.351	-1.58771	1.80
20 in	-1.478	4.680	- .835	- .473	0	-1.48

From Table 16

Table 24
Predicted Values, 7/73 and 1/74, Based on Rate of Change Computations

Nominal Size	Identification (.)	Identification (..)	7/1/73 Predicted Value (.)	7/1/73 Predicted Value (..)	3 S.D. of Predicted Value (.) and (..)	7/1/73 Predicted Sum (.) + (..)	7/1/73 Predicted Difference (.) - (..)	Uncertainty of Sum and Difference
5.000000	M136	H178	29.44	1.59	1.12	31.03	27.86	1.58
6.000000	M115A	H312	29.35	- 3.59	1.30	25.76	32.94	1.84
7.000000	W202A	H105	15.60	- 7.90	1.41	7.70	23.50	2.00
8.000000	M103A	H143	50.60	6.93	1.41	57.53	43.68	2.00
10.000000	M109A	H148	58.64	- 1.72	1.41	56.93	60.36	2.00
12.000000	M135A	H249	72.27	16.17	1.41	88.43	56.10	2.00
16.000000	M109A	H155	67.64	12.60	1.30	80.24	55.05	1.84
20.000000	A157	H146	1.48	19.50	1.24	20.98	-18.02	1.75

Nominal Size	Identification (.)	Identification (..)	1/1/74 Predicted Value (.)	1/1/74 Predicted Value (..)	3 S.D. of Predicted Value (.) and (..)	1/1/74 Predicted Sum (.) + (..)	1/1/74 Predicted Difference (.) - (..)	Uncertainty of Sum and Difference
5.000000	M136	H178	29.59	1.73	1.12	31.32	27.86	1.58
6.000000	M115A	H312	29.43	- 3.97	1.30	25.47	33.40	1.84
7.000000	W202A	H105	15.60	- 8.63	1.41	6.97	24.23	2.00
8.000000	M103A	H143	50.60	6.93	1.41	57.53	43.68	2.00
10.000000	M109A	H148	58.73	- 1.63	1.41	57.10	60.36	2.00
12.000000	M135A	H249	72.27	16.57	1.41	88.84	55.69	2.00
16.000000	M109A	H155	67.75	13.50	1.30	81.25	54.25	1.84
20.000000	A157	H146	.82	18.84	1.24	19.67	-18.02	1.75



URE 20. *Closure, 16((.) — (. .)), Processes I and II.*

All of the rate of change data for the (.) and (. .) blocks and the difference ((.) — (. .)) data are shown in table 23. The Process I estimate of rate of change for the (.) blocks does not agree very well with the historical data. The same is true for the rate of change of the difference from Process I as compared to the measured rate of change of the difference from Process II. Under the assumption that the cause was the small Process I data base, it was decided that the (.) rate of change based on historical values should be retained, and that the best estimate of the rate of change for the (. .) blocks would be that computed from the historical Process I data for the blocks and the measurement Process II data for the differences.

Using the rate of change data from column (1) and column (6) of table 23, new Process I predicted values were determined for 7/73 and 1/74, as shown in table 24. As a typical example, the results for the 16 in blocks, which previously did not close, are shown in figure 20. Clearly the predicted Process II difference is well within the uncertainty of the difference computed from the Process I predicted values using the rate of change data from table 23. The closure is now as expected for the 5 in through the 16 in blocks, as shown in table 25. The closure at the 20 in level is still marginal.

TABLE 25

Summary of Closure
(Predicted July 1973)

Nominal Size	Process (I) Computed Diff.	Uncertainty	Process (II) Predicted Diff.	Uncertainty*
5	27.86	1.58	28.46	.15**
6	32.94	1.84	32.90	.15*
7	23.50	2.00	22.98	.20*
8	43.68	2.00	44.14	.17**
10	60.36	2.00	61.35	.22**
12	56.10	2.00	56.31	.33*
16	55.05	1.84	55.98	.36*
20	-18.02	1.75	-16.46	.36**

* 3 S.D. of predicted difference

** 3 S.D. of mean

At the 20 in level, the greatest confidence at this time is in the Process II((.)−(. .)) by virtue of the number of measurements which have been made. This, however, does not help in establishing the value of either 20(.) or 20(. .). Figure 1 of section 4 indicates only two values prior to establishing the new interferometric process, therefore the historical rate of change of 20(.) is highly subject to question. There were no prior interferometric values for 20(. .). As an expedient action, values for the sum and difference as shown in table 18 were accepted for use in Process II measurements. The uncertainty of the predicted July 1973 values for the sum and difference value at the 20 in level, as shown in table 18, was increased by 2μ in to account for the uncertainty in the rate of change estimates. As a parallel action, additional measurements were started both on the 20(.) and the 20(. .). The result of this is discussed in the next section.

6.5. Process Surveillance

Process surveillance is a continuing operation. In the case of Process I, it is expected that any new value, with appropriate uncertainty, will be in agreement with the current predicted value. Further, each new value adds to the data base used to establish the next predicted value such as was shown in figure 19. If such agreement is obtained, the validity of the predicted value is verified. If such is not the case, either the measurement process or the object has changed. The "out of control" situation requires study to determine what has happened so that necessary actions can be taken to again establish an "in control" situation.

Because of the marginal closure at the 20 in level, a single Process I measurement was made on each of the 20(.) and 20(. .) blocks (measurements of July 9, 1973 in table 27). The difference between the results was ((.)−(. .)) = −16.0 microinches. The estimated uncertainty of this difference is ±2.5 microinches, computed by the relation $(3\sqrt{2})\sigma$ where σ was taken from figure 18. In figure 21 this value is compared with Process II difference measurements, the average Process II difference (from table 20), and the July 1973 Process I predicted difference from table 25, each with appropriate uncertainties. Clearly, the new measurement data alone is not precise enough to resolve the question.

Additional Process I measurements were made on each of the 20 in blocks, and all of the newly determined values were added to the existing data bank. New predicted values, computed differences, and rates of change were determined for these blocks, as summarized in table 26. The new July 1, 1973 predicted difference computed from Process I values, −16.06 microinches, is now in good agreement with the Process II measured difference, −16.45 microinches, as shown in figure 23.On the basis of this analysis, the July 1973 predicted values for the 5 in through 16 in blocks shown in table 24, and for the 20 in blocks, in table 26, were accepted for use as restraints for Process II.

Table 26

Summary 20(.) and (..) Predicted Values

	Previous 7/1/73 Predicted Value	Previous Assumed Slope Estimate	7/1/73 Measured Values	New 7/1/73 Predicted Value	UNC	New Slope Estimate
20(.)	1.48	-1.308	1.3 1.3	1.762	1.24	- .990
20(..)	19.50	-1.308	17.2 18.0	17.822	1.24	-2.387

Summary 7/1/73

Sum = (.) + (..) = 19.58

Diff = (.) − (..) =−16.06 Rate of change 1.397 µin/yr

UNC of Sum and Diff. = 1.8

In the fall of 1973, additional Process I measurements were made on the 5 in through 16 in blocks. Using the expanded data base, as shown in table 27, new estimates of the process standard deviation and the rates of change were made, as shown in table 28. On the basis of the "pooled" standard deviation, 0.642, three blocks, H105–7(. .), H148–10(. .) and H146–20(. .), showed significant rates of change. The data for each block was also analyzed to determine the standard deviation about its "fitted" line. From this it was evident that the s.d.'s are length dependent. The use of the "pooled" s.d. in such a circumstance has the effect of predicting pessimistic uncertainty limits for the shorter

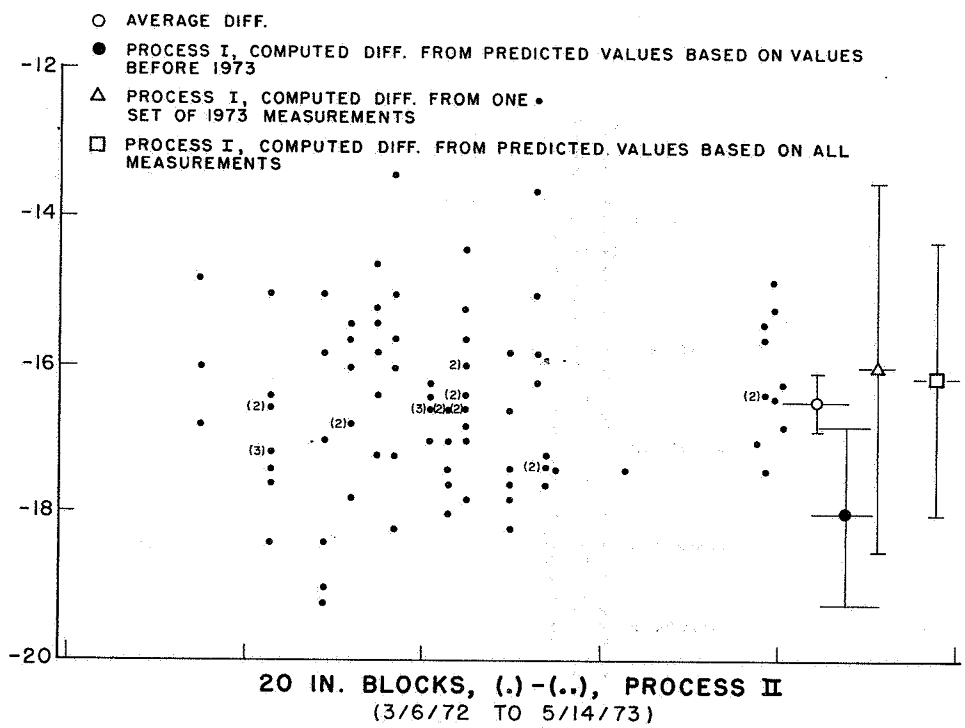


FIGURE 21. *Closure, 20((.) − (. .)), Processes I and II.*

blocks, and optimistic limits for the longer blocks. The process standard deviation was fitted, by the method of least squares, to a line of the form s.d.= βL. Values from this line, as shown in the table 29 column labeled S.D., were used to recheck the rates of change. With the new process precision, one more block, H312–6(. .), indicates a significant rate of change. It is of interest to note that the process standard deviations shown in table 29 are very nearly the same as shown in figure 17, section 6.1.

Using the larger data base and the new process s.d. estimates, predicted values were established for 7/1/73. These values are compared with the values which formed the restraint on Process II in the fall of 1973 in table 30. The agreement is within the combined uncertainty limits. Predicted values, and appropriate Process I restraint data based only on Process I measurements, were established for 1/1/74, and 7/1/74, as shown in table 31. The appropriate Process II data is shown in table 32. The agreement between the differences ((.)−(. .)) as determined by both Process I and Process II is shown in table 33. On January 1, 1974, the values shown were accepted as the restraints on Process II. These values will be used until July 1, 1974, at which time the 7/1/74 value will be used. Additional Process I measurements will be made in the fall of 1974, at which time the values will be "updated," first by checking back to the 7/1/74 values, then by predicting forward for both a six months and a one year period.

The Process II data on the difference (.)−(. .), initially used to establish rates of change, is now used in a different manner. Since the difference between the two reference blocks is determined in every Process II measurement, there is a very large amount of data on the measured differences. In early 1974, all of this data was analyzed to determine a "predicted" measured difference for 1/1/74. The predicted Process II measured difference is compared with the computed Process I difference in table 33. With the exception of the 5 in level, the agreement between the two processes is clearly within the expected limits. The marginal agreement at the 5 in level may indicate that the 0.95 uncertainty of the Process I computed difference is a little optimistic.

TABLE 27. *Process I data through September 1973.*

M136	5.00000000	8	24	71	.18	29.12	.44729495	13.6411	29.30
		10	18	71	.12	29.08	.40707047	13.7890	29.20
		3	6	72	-.17	28.97	.32709971	14.1808	28.80
		3	7	72	.13	28.97	.32672425	14.1836	29.10
		3	8	72	-.37	28.97	.32635185	14.1863	28.60
		9	5	73	.05	28.55	.55864410	15.6712	28.60
		9	7	73	.05	28.55	.56043653	15.6767	28.60
H178	5.00000000	9	13	71	.00	1.20	.38671374	13.6932	1.20
		9	30	71	-.38	1.18	.37524648	13.7397	.80
		10	15	71	.03	1.17	.36551706	13.7808	1.20
		3	6	72	-.25	1.05	.29674699	14.1808	.80
		3	7	72	.25	1.05	.29648054	14.1836	1.30
		3	8	72	.45	1.05	.29621738	14.1863	1.50
		9	5	73	.10	.60	.55793002	15.6712	.70
		9	7	73	-.20	.60	.55968714	15.6767	.40
M115A	6.00000000	8	25	71	.24	29.16	.57410409	13.6438	29.40
		10	18	71	.05	29.15	.52578718	13.7890	29.20
		3	15	72	-.50	29.10	.42056881	14.2055	28.60
		3	16	72	.10	29.10	.42009815	14.2082	29.20
		8	29	73	.05	28.95	.66422637	15.6548	29.00
		8	30	73	.05	28.95	.66527818	15.6575	29.00
H312	6.00000000	9	14	71	.64	-2.24	.55661485	13.6959	-1.60
		9	29	71	-.33	-2.27	.54290299	13.7370	-2.60
		3	15	72	-.62	-2.58	.42063475	14.2055	-3.20
		3	16	72	.19	-2.59	.42016304	14.2082	-2.40
		8	29	73	-.04	-3.56	.66474203	15.6548	-3.60
		8	30	73	.16	-3.56	.66579553	15.6575	-3.40
W202A	7.00000000	8	26	71	-.23	15.73	.84005621	13.6466	15.50
		4	12	72	-.18	15.48	.57447319	14.2795	15.30
		4	13	72	.52	15.48	.57367142	14.2822	16.00
		8	21	73	.05	14.95	.78495288	15.6329	15.00
		8	22	73	-.15	14.95	.78627992	15.6356	14.80
H105	7.00000000	9	15	71	.15	-5.35	.67490634	13.6986	-5.20
		9	28	71	-1.00	-5.40	.66014296	13.7342	-6.40
		4	12	72	.40	-6.20	.48987276	14.2795	-5.80
		4	13	72	.80	-6.20	.48938058	14.2822	-5.40
		8	21	73	-.32	-8.18	.78308550	15.6329	-8.50
		8	22	73	-.02	-8.18	.78436027	15.6356	-8.20
M103A	8.00000000	9	3	71	-.09	50.69	.93686561	13.6658	50.60
		4	5	72	-.33	50.53	.65594131	14.2603	50.20
		4	6	72	.47	50.53	.65500934	14.2630	51.00
		8	23	73	.13	50.17	.88820929	15.6384	50.30
		8	28	73	-.17	50.17	.89569569	15.6521	50.00
H143	8.00000000	9	10	71	1.19	7.11	.76266827	13.6849	8.30
		9	27	71	-.69	7.09	.74118396	13.7315	6.40
		4	5	72	-.37	6.87	.55647095	14.2603	6.50
		4	6	72	-.37	6.87	.55590144	14.2630	6.50
		8	23	73	.31	6.29	.88482391	15.6384	6.60
		8	28	73	-.08	6.28	.89195498	15.6521	6.20
M109A	10.00000000	9	1	71	-.06	58.46	1.20521970	13.6603	58.40
		4	21	72	-.55	58.35	.81265757	14.3041	57.80
		4	24	72	.65	58.35	.80924487	14.3123	59.00
		8	15	73	-.12	58.12	1.11280546	15.6164	58.00
		8	16	73	.08	58.12	1.11472857	15.6192	58.20
H148	10.00000000	9	9	71	.19	-1.59	.97305609	13.6822	-1.40
		9	26	71	.34	-1.64	.94512025	13.7288	-1.30
		4	21	72	-.59	-2.21	.69053131	14.3041	-2.80
		4	24	72	-.18	-2.22	.68860428	14.3123	-2.40
		8	15	73	-.08	-3.52	1.10762404	15.6164	-3.60
		8	16	73	.32	-3.52	1.10944539	15.6192	-3.20
M135A	12.00000000	8	31	71	-.61	72.41	1.49285778	13.6603	71.80
		5	17	72	.78	72.02	.95940517	14.3753	72.80
		5	18	72	.19	72.01	.95811283	14.3781	72.20
		8	13	73	.07	71.33	1.32885112	15.6110	71.40
		8	14	73	-.43	71.33	1.33119936	15.6137	70.90
H249	12.00000000	9	8	71	-.74	15.04	1.19357648	13.6795	14.30
		9	24	71	.57	15.03	1.16109239	13.7233	15.60
		5	17	72	-.06	14.86	.81827080	14.3753	14.80
		5	18	72	.34	14.86	.81763773	14.3781	15.20
		8	13	73	.44	14.56	1.32066737	15.6110	15.00
		8	14	73	-.56	14.56	1.32286757	15.6137	14.00
M109A	16.00000000	8	27	71	.15	67.65	1.56581940	13.6493	67.80
		8	28	71	.05	67.65	1.56306913	13.6521	67.70
		4	28	72	-1.26	67.06	1.08108890	14.3233	65.80
		5	3	72	.95	67.05	1.07705037	14.3370	68.00
		7	12	73	.10	66.00	1.74337101	15.5260	66.10
		7	13	73	.00	66.00	1.74633643	15.5288	66.00
H155	16.00000000	9	7	71	.19	10.01	1.57265292	13.6767	10.20
		9	22	71	.39	10.01	1.53078017	13.7178	10.40
		4	28	72	-1.39	9.99	1.08747844	14.3233	8.60
		5	3	72	.51	9.99	1.08290522	14.3370	10.50
		7	12	73	.05	9.95	1.75076166	15.5260	10.00
		7	13	73	.25	9.95	1.75380018	15.5288	10.20
A157	20.00000000	8	30	71	-2.12	3.52	2.12566841	13.6575	1.40
		3	12	72	1.59	3.01	1.47846642	14.1973	4.60
		3	13	72	.80	3.00	1.47619784	14.2000	3.80
		3	20	72	.61	2.99	1.46071611	14.2192	3.60
		7	9	73	-.44	1.74	2.22950065	15.5178	1.30
		7	11	73	-.44	1.74	2.23770985	15.5233	1.30
H146	20.00000000	9	2	71	-1.00	22.20	1.79572752	13.6630	21.20
		9	23	71	.24	22.06	1.72537805	13.7205	22.30
		3	12	72	.48	20.92	1.28946503	14.1973	21.40
		3	13	72	.18	20.92	1.28801192	14.2000	21.10
		3	20	72	.43	20.87	1.27827047	14.2192	21.30
		7	9	73	-.57	17.77	2.21398526	15.5178	17.20
		7	11	73	.24	17.76	2.22176322	15.5233	18.00

TABLE 28. *Process I standard deviation and block rate of change computations through September 1973.*

See Table 15 for meaning of column headings.

GROUP 7

SER NO	NOMINAL	S.D.	N	XBAR	YBAR	A	B	SDB	T=B/SDB	AVG S.D.
M136	5.00000000	.21579567	7	14.47553778	28.88571358	32.93223906	-.27954235	.10411625	-2.68490598	
H178	5.00000000	.29620085	8	14.38904083	.98749999	5.29873133	-.29961910	.13656049	-2.19403940	
M115A	6.00000000	.28582316	6	14.52648365	29.06666660	30.61824822	-.10681056	.14151073	-.75478770	
H312	6.00000000	.49049272	6	14.52648365	-2.79999995	6.96120870	-.67195950	.24313156	-2.76376915	
M202A	7.00000000	.35611541	5	14.69534218	15.31999981	21.04999971	-.38991947	.19886784	-1.96069641	
H105	7.00000000	.69405091	6	14.54383540	-6.58333325	14.69904518	-1.46332642	.35207165	-4.15633130**	
M103A	8.00000000	.35695536	5	14.69589019	50.41999960	54.29846048	-.26391467	.19828350	-1.33099663	
H143	8.00000000	.75277828	6	14.53835595	6.74999988	12.88094354	-.42170819	.37720064	-1.11799434	
M109A	10.00000000	.50056331	5	14.70246542	58.27999926	60.88251352	-.17701210	.28555798	-.61988149	
H148	10.00000000	.40004947	6	14.54383528	-2.44999996	12.03702903	-.99609414	.20455815	-4.86949134**	
M135A	12.00000000	.63595785	5	14.72767103	71.81999874	80.01925182	-.55672438	.37021084	-1.50380355	
H249	12.00000000	.61148902	6	14.56347001	14.81666660	18.44053245	-.24883259	.31546181	-.78878830	
M109A	16.00000000	.79493372	6	14.50273943	66.89999962	79.65187836	-.87927383	.41829779	-2.10202837	
H155	16.00000000	.78185679	6	14.51826453	9.98333323	10.42036641	-.03010230	.42054037	-.07158006	
A157	20.00000000	1.45303530	6	14.55251098	2.66666663	16.61056948	-.95817848	.83381634	-1.14914811	
H146	20.00000000	.61379633	7	14.43444192	20.35714245	54.82744598	-2.38805932	.32253424	-7.40404904**	

TABLE 29. *Accepted Process I standard deviation, January 15, 1974.*

See Table 15 for meanings of column headings.

SER NO	NOMINAL	S.D.	N	XBAR	YBAR	A	B	SDB	T=B/SDB	AVG S.D.
M136	5.00000000	.27000000	7	14.47553778	28.68571358	32.93223906	-.27954235	.13026854	-2.14589289	
H178	5.00000000	.27000000	8	14.38904083	.98749999	5.29873133	-.29961910	.12448085	-2.40694937	
M115A	6.00000000	.32000000	6	14.52648365	29.06666660	30.61824822	-.10681056	.15843165	-.67417440	
H312	6.00000000	.32000000	6	14.52648365	-2.79999995	6.96120870	-.67195950	.15862029	-4.23627704**	
M202A	7.00000000	.38000000	5	14.69534218	15.31999981	21.04999971	-.38991947	.21220587	-1.83745846	
H105	7.00000000	.38000000	6	14.54383540	-6.58333325	14.69904518	-1.46332642	.19276284	-7.59133035**	
M103A	8.00000000	.43000000	5	14.69589019	50.41999960	54.29846048	-.26391467	.23885873	-1.10489857	
H143	8.00000000	.43000000	6	14.53835595	6.74999988	12.88094354	-.42170819	.21546355	-1.95721361	
M109A	10.00000000	.54000000	5	14.70246542	58.27999926	60.88251352	-.17701210	.30805555	-.57461098	
H148	10.00000000	.54000000	6	14.54383528	-2.44999996	12.03702903	-.99609414	.27611935	-3.60747671**	
M135A	12.00000000	.65000000	5	14.72767103	71.81999874	80.01925182	-.55672438	.37838521	-1.47131643	
H249	12.00000000	.65000000	6	14.56347001	14.81666660	18.44053245	-.24883259	.33532929	-.74205443	
M109A	16.00000000	.86000000	6	14.50273943	66.89999962	79.65187836	-.87927383	.45253596	-1.94299215	
H155	16.00000000	.86000000	6	14.51826453	9.98333323	10.42036641	-.03010230	.46257156	-.06507599	
A157	20.00000000	1.08000000	6	14.55251098	2.66666663	16.61056948	-.95817848	.61975208	-1.54606739	
H146	20.00000000	1.08000000	7	14.43444192	20.35714245	54.82744598	-2.38805932	.56751231	-4.20794272**	
										.6292269

Table 30

Comparison of 7/1/73 Predicted Values

Serial Number	Nominal Size	Predicted[1] Value (71-72 Data)	3 S.D.	Predicted[2] Value (71-73 Data)	3 S.D.	Delta Predicted Value
136	5.0	29.44	1.11	28.60	.67	.84
178	5.0	1.59	1.11	.66	.67	.93
115	6.0	29.35	1.30	28.96	.80	.39
312	6.0	- 3.59	1.30	- 3.45	.80	- .14
202	7.0	15.60	1.41	15.01	.98	.59
105	7.0	- 7.90	1.41	- 7.98	.96	.08
103	8.0	50.60	1.41	50.21	1.10	.39
143	8.0	6.93	1.41	6.35	1.08	.38
109	10.0	58.64	1.41	58.14	1.40	.50
148	10.0	- 1.72	1.41	- 3.40	1.38	1.68
135	12.0	72.27	1.41	71.39	1.69	.88
249	12.0	16.17	1.41	14.58	1.65	1.56
109	16.0	67.64	1.30	66.03	2.29	1.61
155	16.0	12.60	1.30	9.95	2.31	2.65
157	20.0	1.48	1.24	1.76	3.00	- .38
146	20.0	19.50	1.24	17.82	2.94	1.68

[1] Based on 71-72 Process I Data Supplemented with Process II (.)-(..) Data

[2] Predicted Values Based on 71-73 Process I Data

TABLE 31. *Accepted Process I data for (.) and (. .) reference blocks, January and July 1974.*

NOM. SIZE	IDENTIFICATION (.)	(..)	1- 1-74 PRED VAL (.)	(..)	3SIG PRED VAL (.)	(..)	1- 1-74 PRED SUM (.)+(..)	1- 1-74 PRED DIFF (.)-(..)	UNC OF SUM
5.000000	M136	H178	28.46	.50	.67	.67	28.96	27.95	.95
6.000000	M115A	H312	28.91	-3.79	.80	.80	25.12	32.70	1.14
7.000000	W202A	H105	14.81	-8.72	.98	.96	6.09	23.53	1.37
8.000000	M103A	H143	50.08	6.13	1.10	1.08	56.21	43.94	1.54
10.000000	M109A	H148	58.05	-3.90	1.40	1.38	54.15	61.95	1.97
12.000000	M135A	H249	71.11	14.46	1.69	1.65	85.57	56.65	2.36
16.000000	M109A	H155	65.58	9.94	2.29	2.31	75.52	55.64	3.26
20.000000	A157	H146	1.28	16.61	3.00	2.94	17.89	-15.33	4.20

NOM. SIZE	IDENTIFICATION (.)	(..)	7- 1-74 PRED VAL (.)	(..)	3SIG PRED VAL (.)	(..)	7- 1-74 PRED SUM (.)+(..)	7- 1-74 PRED DIFF (.)-(..)	UNC OF SUM
5.000000	M136	H178	28.32	.36	.85	.84	28.68	27.96	1.19
6.000000	M115A	H312	28.86	-4.12	1.01	1.02	24.73	32.98	1.44
7.000000	W202A	H105	14.62	-9.44	1.25	1.22	5.18	24.06	1.75
8.000000	M103A	H143	49.94	5.92	1.41	1.37	55.87	44.02	1.97
10.000000	M109A	H148	57.96	-4.39	1.81	1.75	53.57	62.36	2.51
12.000000	M135A	H249	70.84	14.34	2.19	2.10	85.17	56.50	3.03
16.000000	M109A	H155	65.15	9.92	2.90	2.94	75.07	55.22	4.13
20.000000	A157	H146	.80	15.43	3.85	3.72	16.24	-14.63	5.35

TABLE 32. *Accepted Process II data for (.) and (..) reference blocks. January 1974.*

VALUES (MICROINCHES) FOR REFERENCE BLOCKS AND PROCESS PARAMETERS AS OF FEB 74
FOR USE IN CALIBRATION OF TEST BLOCKS BY MECHANICAL INTERCOMPARISON

LENGTH	IDENTIFICATION (.)	(..)	RESTRAINT (.)+(..)	UNCERTAINTY	CHECK STANDARD (.)-(..)	S.D. (WITHIN)	S.D. (TOTAL)
5.000000	M136	H178	28.963	.946	28.460	.47	.54
6.000000	M115A	H312	25.117	1.137	33.476	.47	.59
7.000000	W202A	H105	6.092	1.372	23.562	.47	.63
8.000000	M103A	H143	56.208	1.544	44.212	.47	.67
10.000000	M109A	H148	54.147	1.966	61.373	.47	.76
12.000000	M135A	H249	85.569	2.363	56.908	.47	.85
16.000000	M109A	H155	75.520	3.257	57.282	.57	1.02
20.000000	A157	H146	17.889	4.201	-16.484	.72	1.20

Table 33

Comparison ((.)-(..), Process I and Process II, January 1974

Nominal Size	Process I (.)-(..)	UNC	Process II (For 1-1-74) (.)-(..)	Total S.D.	D.F.	3S.D. Mean	Δ
5	27.95	.95	28.460	.489	90	.153	-.51
6	32.70	1.14	33.476	.525	97	.513	- .78
7	23.53	1.37	23.562	.543	106	.546	- .03
8	43.94	1.54	44.212	.753	309	.129	- .27
10	61.95	1.97	61.373	.729	88	.231	.58
12	56.65	2.36	56.908	1.104	89	.348	- .26
16	55.64	3.26	57.282	1.072	71	.378	-1.24
20	-15.33	4.20	-16.484	1.054	94	.324	1.15

Δ is the difference between computed Process I((.)-(..)) and the measured Process II((.)-(..)).

7 Measurement Process in Action

7.1. Uncertainty of the Transferred Value

The concept of a measurement system requires that values assigned to represent certain characteristics of objects be reasonably unique and repeatable over time and changes in location. It is expected that sequences of measurements of the same thing made at various times and at different locations show evidence of convergence to the same limiting mean. Uncertainty statements are, in essence, predictors of the degree to which such closure can be attained. Failure to agree within uncertainty limits is an indication that the two processes are fundamentally different, or that the uncertainty statement does not adequately describe the error bounds. For a practical measurement, the measurement algorithm, or the mathematical model of the measurement process, cannot possibly reflect all of the sources of variability. The instrument or comparator cannot differentiate between a real change and all of the perturbations which change the indication in the same manner as a change in the object. Nonetheless, it is important to know the bounds for the variability which occurs in the course of making measurements. Redundancy, either by repeated measurements or incorporated in a particular measurement process, provides a means for assessing this variability.

In order to illustrate the nature of a realistic uncertainty statement, consider first the collection of simulated measurement data in figure 22. The data shown reflects the effects of variability from four cyclic sources over the time period necessary for 300 measurements. This data has the appearance of coming from a reasonably well behaved measurement process. There are no apparent trends and there is little evidence of grouping. The 3 s.d. limits appear to be bounds for process variability. One would surely expect the next measurement result to be within these prescribed limits. Further, if the next measurement was defined to be the average of n independent measurements, one would expect this average to agree with the average shown within 3(s.d./($\sqrt{n}$). Without knowledge of independent parameters which are proportional to the magnitude of each source of variability, there is no way to further analyze this data. The random component of the uncertainty of the result would be a function of the s.d. and the definition of the result (i.e., single measurement, or the average of n measurements).

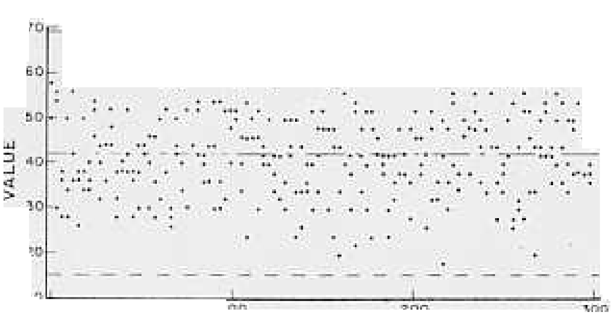


FIGURE 22. *Simulated measurement data.*

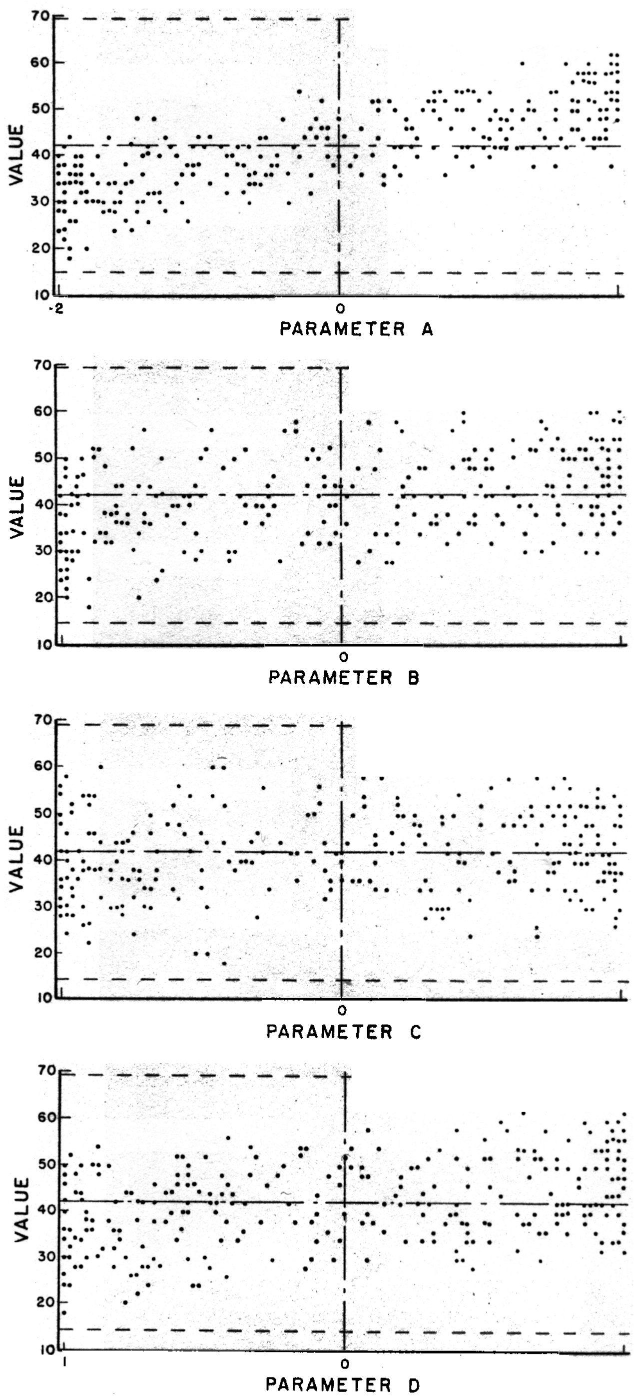


FIGURE 23. *Simulated measurement data analysis.*

In this process simulation, there are identifiable parameters which are proportional to the effects of sources contributing to the process variability. Recording the parameter values along with each measurement result permits the use of correlation studies to further evaluate the process. In figure 23, the parameter for each source of variability is plotted against the appropriate measurement result. For parameters B, C, and D, there is little evidence of correlation. While the variability of these parameters contributes to the process variability, one cannot differentiate between their respective contributions. Clearly, there is a correlation with parameter A. This correlation indicates that a "between time" variability associated with parameter A is influencing the measurement results. The effect is systematic, that is, the result is high when the parameter value is high, and vice versa. It should be noted, however, that in spite of the existence of the systematic effect, the initial $3\sigma_T$ limit is still an appropriate bound for the process variability.

If, over the sequence of the 300 measurements, the variability of the result reflects the maximum excursion of each parameter in this and all similar measurement processes, the initial s.d. is an appropriate basis for a realistic uncertainty statement. This includes one parameter frequently overlooked, a change in location. The variability of a given parameter in one facility, such as air density, may be only a small fraction of the variability of the same parameter over all locations. Other parameters may be related only to changes in location. If, under the conditions stated above, the performance of the process is adequate for the intended use of the results, there would be no reason for change. On the other hand, having identified the source of variability, action can be taken to reduce the magnitude of the systematic effect with a resulting decrease in process s.d. as shown in figure 24.

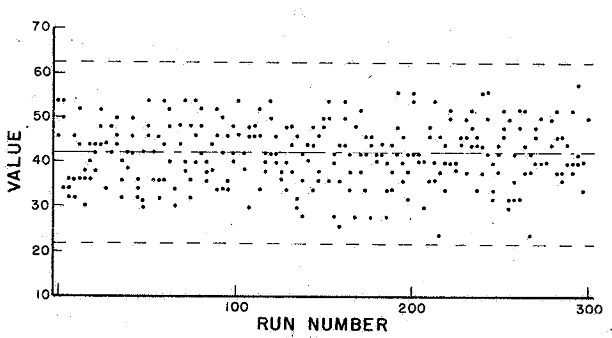


FIGURE 24. *Simulated measurement data—systematic effect minimized.*

The two measurement processes used to assign values to "working" gage blocks are described by the above simulation. Process II, the comparison, is like the process described by figure 22. Each value from Process II is the result of a sequence of measurements over a short time span (about 5 minutes) so that conditions do not change very much. One would normally expect that the standard deviation of the collection of repeated measurements, (.)−(. .), would be a function of the redundancy of the design and the within-group standard deviation. The design solution gives values for the difference between the "knowns," ((.)−(. .)), and the "unknowns," (x) and (y). These values are linear combinations of "single" measurements so that:

$$\text{s.d.}\left((.)-(.\,.)\right)=A\sigma_w=\sigma_T$$
$$\text{s.d.}(x)=\text{s.d.}(y)=B\sigma_w$$

where A, B, reflect the redundancy of the design. For the designs used, $A\approx 0.58$, so that one would expect $\sigma_T<\sigma_w$. Table 32 showed clearly that this is not the case thus indicating that there is a long term source of variability which, as yet, has not been characterized. For Process II, the random component of the uncertainty must reflect this variability.[18]

Process I is like the process described by figure 23. There are at least four independent parameters associated with known sources of variability, the atmospheric pressure, temperature and relative humidity, and the temperature of the block. As an example of correlation, figure 25 shows clearly that the initial variability associated with the values obtained for the 10 in cervit control block, in figure 9, is related to relative humidity. The corrective action taken, described in reference [14], resulted in a smaller standard deviation. As collections of data increase, additional correlation studies provide insight as to process behavior, and provide a means to identify and reduce the magnitude of systematic variability.

The values established in Process I are used as "constants" in the restraint for Process II, thus the uncertainty of the restraint, (.)+(. .), is in part the systematic error associated with Process II results. The uncertainty of the restraint, Process I being free from known sources of systematic error, is three times the combined standard deviations associated with the (.) and (. .) predicted values such as shown in table 32. A proportion part, N/R, where N is the nominal value of the restraint and R is the nominal value of the "unknown," becomes the systematic error term for the result from Process II. For the restraint used in the current designs, the S.E. component of the uncertainty of the announced value for the "unknowns" is (½)(Unc.(.)+(. .)). The (Unc.(.)+(. .)) is based on Process I performance parameters. For the design in current use, the random com-

[18] With the exception of environment temperature, there are no independent parameters at the present time which can be used to identify the source or sources of this systematic variability in Process II. The metrologist is interested in determining the source and magnitude of the between-time components of variability. Understanding the nature of this variability generally leads to improved equipment and measurement procedures. On the other hand, one may not be able to reduce the magnitude of the variability without severely limiting the practicality of the measurement process.

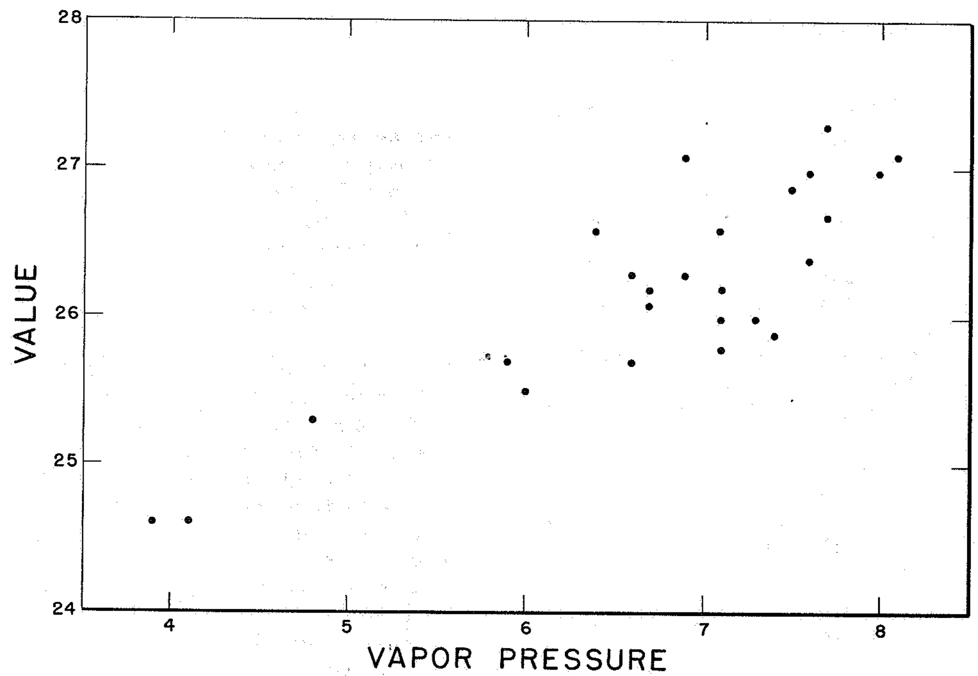


FIGURE 25. *Value versus vapor pressure.*

ponent of the uncertainty of the announced values is computed by the relation:

$$3\sigma(x) = 3\sigma(y) = 3\sqrt{\sigma_T^2 - \frac{7}{48}\sigma_w^2}$$

as given in section 10 of reference [6]. The total s.d., σ_T, and the within s.d., σ_w, are Process II performance parameters.

7.2. Verifying the Uncertainty

Having established the appropriate quantitative process performance parameter and constructed a suitable uncertainty statement, it is of interest to verify that the statement is in fact descriptive of the expected closure. For one such test, a set of reference blocks designated NBS (. . .) was considered as a typical set of long blocks. The results of measurements on these blocks are shown in table 34. The first sequence of measurements, labeled "typical," established values for these blocks relative to the values of the NBS (.) and (. .) reference blocks by means of Process II. The (. . .) blocks were also included in many sequences of measurements in the course of evaluating Process II. The average of the collection of values is also shown. Finally, values were assigned to the (. . .) blocks by Process I measurements. The uncertainties shown for each of the three values are based on the latest process information. In all cases, the values agree within the expected limits.

As a second test, in another facility the values assigned to reference block set No. 496 were compared to reference block set S–183873 [22]. The values for blocks of set No. 496 were assigned by a normal Process II measurement. (The Process II measurements have been used for some time in the normal NBS gage block calibration service.) The values for the blocks of Set No. S–183873 had been assigned by the old multiple wavelength NBS interferometric process. Under the assumption that the initial uncertainties of the S–183873 values were reasonably correct, the closure, as shown in table 35, was within limits established by the uncertainty of the Process II measurements and the uncertainty of the values assigned to Set No. S–183873.

7.3. Other Measurement Processes

Measurement processes in many different environments comprise a measurement system. Consistency within the system is assured if closure, within the capabilities of the various processes, can be demonstrated. Early in the program, a cooperative effort was made to verify closure between two different measurement processes. Blocks of nominal size 5 in and 16 in were chosen for this work. The comparison designs used required a comparison of all pairs in a group of four, (six observed differences). One block was considered as a "known." The second block was considered as a "check standard," and the other two blocks were considered to be "unknowns." In each case, two blocks went

Table 34

Summary Data for (...) Reference Blocks

Designation (...)	PROCESS II								PROCESS I		
	Typical		Summary								
	Date	Y_I	Ave	No	S.D.*	S.E.	UNC**		Ave	No	UNC
1117- 5	2/22/72	17.70	17.5	40	.54	.47	.73		16.9	3	.47
1324- 6	2/24/72	-6.37	-7.3	83	.59	.57	.76		-8.7	2	.79
1136- 7	2/25/72	12.03	12.0	40	.63	.69	.98		12.7	2	.81
1140- 8	2/28/72	8.80	8.3	98	.67	.77	.97		9.1	3	.75
1103-10	2/29/72	22.20	21.8	36	.76	.98	1.37		21.2	2	1.15
1132-12	3/1/72	9.13	9.6	40	.85	1.18	1.60		10.6	2	1.38
1134-16	3/3/72	10.00	9.3	17	1.02	1.63	2.37		10.2	2	1.82
1123-20	3/6/72	43.00	42.8	25	1.20	2.10	2.82		46.2	3	1.87

* Process total S.D. from Table 32.

** UNC = 3(total S.D.)/$\sqrt{n}$ + S.E. (S.E. is one-half of the uncertainty of the sum in Table 23.

Table 35

Closure on Set No. 496

Nominal Size	NBS Process II					Lab "A"		Δ
	$\bar{Y}_{II}$*	S.D.	n	S.E.	UNC	Y_{II}**	S.E.	
5	0.8	.54	1	.48	2.10	0.4	3	0.4
6	9.0	.59	1	.57	2.34	13.5	3	-4.5
7	18.5	.63	1	.68	2.57	19.1	3	-0.6
8	13.0	.67	1	.77	2.78	11.4	3	1.6
10	15.4	.76	1	.99	3.27	16.8	3	-1.4
12	19.6	.85	1	1.18	3.73	21.8	5	-2.2
16	19.2	1.02	2	1.63	2.35	19.3	5	-0.1
20	12.7	1.20	1	2.10	5.60	19.6	5	-6.9

* Average of n values with respect to NBS(.) and (..) blocks.

** With reference to a set S-183873 (values previously established by multiple wavelength interferometry with estimated uncertainty, S.E., as shown).

Δ is the difference between the $\bar{Y}_{II}$* values and the Y_{II}** values.

Table 36

"Echelon" Closure, Process II

Location	Block Serial No.							
	M136	H178	4114	X616A + X368A	A138	A142	I290	I162
NBS n=6	28.0 — R	3.0 (.62) C	3.4 (.62)	8.3 (.62)				
Lab "A" n=3		3.0 — R	3.1 (1.62) C		-22.5 (1.62)	-12.3 (1.62)		
Lab "B" n=3					-22.5 — R	-10.2 (2.62) C	3.0 (2.62)	.8 (2.62)
NBS NBS n=1	28.0 — R	2.0 (1.4) C			-22.1 (1.4)	-11.8 (1.4)		
	28.0 — R	2.8 (1.4) C					1.6 (1.4)	-.7 (1.4)

( ) Uncertainty
R Restraint Value
C "Check Standard"

to the next measurement process, assuming the same roles. The results of the work on the 16 in blocks are discussed in section 5.5, because of thermal problems encountered. The results of the work with the 5 in blocks are shown in table 36.

Referring to table 36, five independent sequences of measurements were made at NBS on the blocks designated M136, H178, 4114, and (X616A + X368A). M136 was assumed known and without error, so that the value assigned was the restraint on the solution for values for the remaining blocks. The values shown for the other blocks is the average of 5 independent measurements. The uncertainties shown are those thought to be appropriate at the time. Two blocks, H178 and 4114, were forwarded to "Lab A."[19]

Three independent sequences of measurements were made at "Lab A," using the assigned value for H178 as the restraint, and 4114 as a "check standard." Blocks A138 and A142 were the "unknowns." The agreement between the value obtained for 4114 relative to H178, and the "known" value of 4114 relative to M136 is one measure of the agreement between the two measurement processes. Blocks A138 and A142 were passed on to a simulated "Lab B." The initial uncertainty of the value for H178, unc. = 0.62 microinches, was the systematic component of the "Lab A" uncertainty.

Again, three independent sequences of measurements were made in "Lab B," using the value assigned to A136 by "Lab A" as the restraint. The agreement between the value assigned to A142 relative to A138, and the "known" value for A142 is a measure of the agreement between "Lab A" and "Lab B." In normal procedures, blocks I290 and I162 would pass on to other labs. In order to close out the test, blocks H178, A138, A142, I290 and I162 were returned to NBS. Using the same sequence of comparisons, the pairs of blocks, A138 and A142, and I290 and I162, were each compared once with the pair M136 and H178. Again, the values obtained for H178 relative to M136 verified the consistency of the NBS Process. The values obtained for the other blocks demonstrated closure through one and two transfer processes. In all cases the agreement was within the expected limits.

[19] The U.S. Navy Eastern Standards Laboratory assumed the role of both "Lab A" and "Lab B" in this study.

In most metrology laboratories, long gage block comparisons are made using equipment designed specifically for the purpose. In use, however, a wide variety of equipment could be used to make the necessary comparisons. In order to simulate a situation in which one has to construct a comparator, the arrangement shown in figure 26 was used to compare 5 in and 12 in blocks. Six independent measurements were made, following an intercomparison design. The results are compared with both previous work and an average of two direct comparisons in table 37. With the particular equipment used, the standard deviation of the simplified "comparator" was 2 microinches. Assuming the uncertainty of the ((.) + USN) restraint is about the same as the uncertainty of the ((.)+(..)) restraint, the closure obtained was within the expected limits.

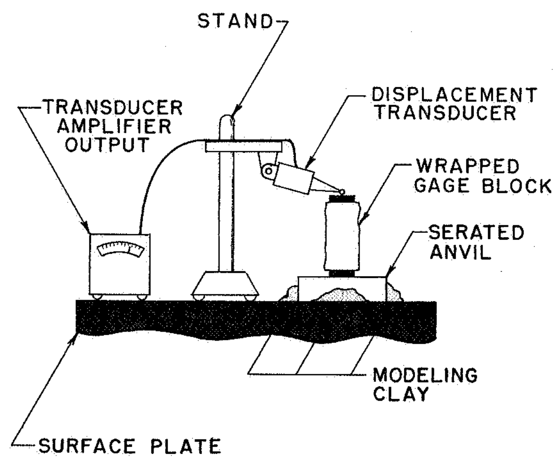


FIGURE 26. *Schematic diagram, simplified comparator.*

TABLE 37

Comparison, Process II Results with Results from Simplified Comparison Process

Date	Type of Measurement	Nominal Size	Restraint Block	Unknowns				Estimated Uncertainty		
				Block	Value	Block	Value	Standard Deviation	S.E.*	Total
2/73	6 Series with process as shown in figure 26	5	(.)+(..)	4114	- .9	1628	57.13	2.0	.95	3.4
12/70	Average of two direct comparisons	5	(.)	-	-	1628	57	.5	.67	1.72
11/71	6 Series with regular process	5	(.)+USN	4114	-1.6	-	-	.5	.95	2.45
12/70	Average of two direct comparisons	12	(.)	-	-		118	.8	1.69	3.1
9/71	6 Series with regular process	12	(.)+USN	3507	16	-	-	.8	2.36	2.69
2/73	6 Series with process as shown in figure 26	12	(.)+(..)	3507	22		120	2.0	2.36	4.81

* The systematic error, or uncertainty, associated with the restraint block values.

7.4. Operational "Addition"

A recurring question in the use of all gage blocks is associated with "additivity." Gage blocks of various selected nominal sizes are "wrung" together to construct lengths which are not normally assigned to single blocks. The "wringing" is in effect an "operational" definition of length addition. One is concerned as to the agreement between the length of the combination, and the sum of the lengths assigned to each block in the combination. The element of interest is the variability of the thickness of the film in the interfaces between the blocks.

The mechanism of "wringing" is not well understood. One can, however, postulate the thickness of the film as being somewhere between "zero" for bare metal contact, such as discussed in section 5.6, and a "max" associated with the "feel" of the "wring." Each Process I measurement includes a finite film thickness, that between the block and the platen to which it is "wrung." The average of a collection of repeated Process I measurements includes an average film thickness. The variability of the collection includes the variability of the film thickness about that average. A comparison between the total process s.d., σ_T, and the s.d. associated with the collection of values for the control blocks, which are "permanently wrung" to their respective platens, should provide a measure of the "wringing film" variability associated with the NBS artifacts and procedures.

For Process I measurements at the 10 in level, $\sigma_T = 0.54$, from table 32 in section 6.5. For the collection of Process I values for the 10 in control block, part of which are shown in figure 9 in section 5.1, s.d. = 0.33. One can think of the "wringing" film variability as being a between-time component of variance which is step-like in nature. For each "wring," there is a reasonably stable film of some finite thickness, the thickness varying about some average thickness. Under these conditions, the variability of the film, σ_f, is $((0.54)^2-(0.33)^2)^{1/2}$ or $\sigma_f = 0.43$. Thus, on the average, one would expect the "wringing" film thickness to be on the order of 2 or $3\sigma_f$. The expected Process I total s.d. for a "wrung" combination of blocks would be $\sigma_T = ((0.33)^2 + n(.43)^2)^{1/2}$, or for a combination of two blocks, $\sigma_T = 0.69$.

In the first series of measurements reported in table 36, one 5 in "block" was a summation of a 2 in block and a 3 in block. Between each of the six series of intercomparisons, the summation was disassembled, cleaned, and reassembled. A comparison of the values obtained for the summation, and the two single blocks, is shown in figure 27. While all of the values were well within the expected limits based on the total standard deviation of Process II, the values associated with the summation appear to show more variability.

From estimates of the s.d. of the points shown, the variability of the collection of values for the summation has an additional random component of s.d. 0.5 microinches over and above the s.d. of the values for the single blocks. This is in reasonable agreement with σ_f as determined above. The accepted values for the individual blocks were 7.3 and 0.3, thus the sum of the individual values agrees very well with the value assigned to the summation.

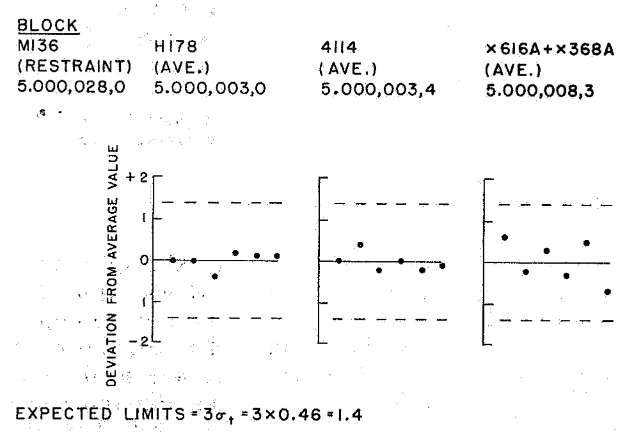


FIGURE 27. *Variability of 5 in combination.*

Process I measurements were made on summations of nominal size 10, 14, 16 and 20 in. The results are summarized in table 38. Again, the values obtained for the summations are in good agreement with the sums of the accepted values of the individual blocks. The accepted values are from table 13. There is some evidence that the correction for the compression of the bottom block of the stack due to the weight of the top block is smaller than the "wringing" variability.

One might conclude from the above data that, with careful "wringing," the variability associated with the "wringing" process is not large. Certainly, all of the evidence seems to support such a conclusion. However, "wringing" is a complex phenomenon. All of the factors which might influence variability from this source have not been identified. The above data merely indicates that the procedures used with long blocks at the NBS do not cause a large variability in the results. "Wringing" to establish stacks of desired length is a common practice with short blocks. Extensions of the above studies are in process in order to establish a quantitative estimate of the limits of variability expected in stacks of 2, 3, and 4 short gage blocks [23].

The introduction to the appendix, as shown in figure 29, states the criteria used to define the state of operation "in control." The body of the appendix, as shown in figure 30, lists the serial numbers of the NBS reference blocks which were used. Four blocks of the same nominal size are used

Table 38

Value of Summation vs Summation of Values

Nominal Size	Block Summation	$\bar{Y}_I$ (Meas.)	n	S.D.	UNC	ΣY_I*	"C"
10	H178 + M136	30.3	3	.54	.94	30.1	.3
14	W202A + H105	9.0	3	(.76)	(1.32)	9.9	.5
16	H143 + M103A	57.9	3	.86	1.49	57.1	.6
20	H148 + M109	54.9	3	1.08	1.87	56.4	.9

* Computed from values shown in table 13, including the compression correction, "C", shown in the next column.

in each comparison sequence. The NBS (.) and (. .) blocks are the reference blocks. Blocks from the set for which this report applies were grouped with the set of blocks mentioned to establish the group of four. The check standard accepted value is the Process II long term average difference between the reference blocks, ((.)−(. .))II. The observed value is that obtained in the sequence of measurements used to establish the value in the report. The t test is based on the accepted total standard deviation for Process II, as shown. The within standard deviation is that associated with the sequence of measurements used to establish the reported values. The accepted standard deviation is the long term average within standard deviation for Process II. Eight sequences of measurements were required to establish the reported values and no repeats were necessary by virtue of the process being "out of control."

8. Summary

The Measurement Assurance Programs emphasize the establishment of confidence in measurement results by operational demonstration. One is concerned with the variability of his own process and the relationship between his results and the task he is trying to accomplish. Process variability over time included effects from all sources, some of which are known or can be deduced, some of which are suspected or imagined, and some of which are not as yet or may never be detected. Realistic error limits, or bounds for the effects of systematic errors, provide both a means to assess the results and a basis for monitoring the process performance. The work described is concerned with relating the generally unaccessible defined length unit, in terms of wavelength, to accessible artifacts such as gage blocks. The basic techniques which have been utilized are precise process definition, redundancy of measurement over time and location, and closure both between the results from different processes and between predicted and observed values. In essence, one searches for a measurement algorithm which adequately describes the observed results.

In detail this paper documents the transition from multiple wavelength interferometry to single wavelength interferometry in the assignment of length values to long gage blocks, and the development of a suitable transfer process to provide access to the unit. Two aspects of this work emerge with clarity: the benefits of the "one shot" assignment of values to large numbers of gage blocks by interferometry as used in the past were largely esthetic, in addition to being costly and time consuming; and NBS must devote its interferometric measurement capability to the maintenance of suitable reference artifacts, techniques for closure between various interferometric measurement processes, and to the development of large "on-scale" range comparators.

This paper documents in part a measurement process analysis; "in part" because a process study is a continuing effort to understand the measurement process itself. Heretofore, in pursuit of a minimum uncertainty, major efforts were made to severely restrict the previous measurement processes. Meaningful measurements are made in a real world subject to all sorts of perturbations. Realistic uncertainties in this real world direct the efforts toward process definition and process response to these perturbations.

In the work described, the main effort has been to establish realistic uncertainty statements. The present task has been merely to identify, and "correct" if possible, the largest sources of error in the restricted environmental conditions of the NBS facilities. Further efforts are needed to identify other sources of systematic errors which are present, as evidenced by the magnitude of the total standard deviation. Measurements must be made over a wider range of environmental conditions. One should be able to predict a realistic uncertainty for any set of conditions and objects, and then verify the validity of the uncertainty by actual measurement.

DIMENSIONAL TECHNOLOGY
NATIONAL BUREAU OF STANDARDS
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TEST NO. 210448
OPERATOR CT　　COMPARATOR G3
DOALL　　GRADE AA
4/ 2/74

PAGE 6
GROUP 7
5.000000 TO 20.000000

SERIAL NUMBER	LENGTH AT 20 C (INCHES)	= NOMINAL	+ CORR.	UNC.	= SYS. ERROR	+ 3 S.D.	COEFF. OF EXP.
			(V A L U E S	IN M I C R O-I N C H E S)			
4521	5.00000039	5.000000	.394	2.001	.473	1.528	11.5
4703	6.00000838	6.000000	8.384	2.255	.568	1.686	11.5
3316	7.00000808	7.000000	8.075	2.498	.686	1.812	11.5
4002	8.00000962	8.000000	9.625	2.709	.772	1.937	11.5
4014	10.00000412	10.000000	4.124	3.199	.983	2.216	11.5
4012	12.00001455	12.000000	14.551	3.674	1.182	2.493	11.5
3505	16.00002761	16.000000	27.606	4.618	1.628	2.990	11.5
3620	20.00002004	20.000000	20.040	5.605	2.100	3.504	11.5

FIGURE 28. *Report of calibration.*

APPENDIX

THIS APPENDIX PRESENTS DATA ON THE MEASUREMENT PROCESS BY WHICH THE VALUES WERE ASSIGNED TO THE BLOCKS. THE PROCESS FOR EACH NOMINAL SIZE IS VERIFIED AS BEING IN A STATE OF STATISTICAL CONTROL BY USING BOTH THE VALUE OBTAINED FOR THE CHECK STANDARD AND THAT OBTAINED FOR THE STANDARD DEVIATION.

THE STANDARD DEVIATION (BASED ON 4 DEGREES OF FREEDOM) COMPUTED FROM THE DEVIATIONS BETWEEN OBSERVED AND PREDICTED VALUES IS COMPARED BY TAKING ITS RATIO TO THE LONG RUN VALUE FOR THE WITHIN RUN STANDARD DEVIATION. IF THE SQUARE OF THE RATIO DOES NOT EXCEED THE CRITICAL VALUE, 4.62, FOR THE .01 PROBABILITY POINT OF THE F DISTRIBUTION, THE PROCESS IS REGARDED AS BEING IN CONTROL FOR PRECISION. IN ADDITION, THE VALUE FOR THE CHECK STANDARD SHOULD NOT DEVIATE FROM ITS ACCEPTED VALUE BY MORE THAN 3.29 TIMES THE 'TOTAL' STANDARD DEVIATION FOR THE PROCESS TO BE REGARDED AS BEING IN CONTROL WITH RESPECT TO POSSIBLE SYSTEMATIC SHIFTS IN PERFORMANCE. (THE CRITICAL VALUE 3.29 CORRESPONDS TO THE 0.001 PROBABILITY POINT FOR THE STANDARDIZED NORMAL DISTRIBUTION.)

IF EITHER OF THESE TESTS ARE 'FAILED,' THE COMPLETE SET OF MEASUREMENTS FOR THAT NOMINAL LENGTH ARE REPEATED AND THESE INDEPENDENT NEW VALUES ARE USED IN THIS REPORT.

FIGURE 29. *Introduction to report of calibration appendix.*

DIMENSIONAL TECHNOLOGY
NATIONAL BUREAU OF STANDARDS
WASHINGTON D.C. 20234
TEST NO. 210448
OPERATOR CT   COMPARATOR G3
DOALL   GRADE AA
4/ 2/74

PAGE A- 6

GROUP 7
5.000000 TO 20.000000

THESE ARTIFACTS WERE GROUPED WITH SIMILAR ARTIFACTS FROM TEST NO. 210249 IN THE FOLLOWING SERIES OF MEASUREMENTS.

NOMINAL	SERIAL NO. (.)	(..)	CHECK STD. ACC.	OBS.	T	STANDARD DEVIATION THIS RUN (D.F.=4)	ACC. VALUE	F
5.000000	M136	H178	28.460	29.125	1.231	.185	.470	.155
6.000000	M115A	H312	33.476	33.583	.182	.190	.470	.164
7.000000	W202A	H105	23.562	23.692	.206	.330	.470	.493
8.000000	M103A	H143	44.212	44.875	.990	.541	.470	1.324
10.000000	M109A	H148	61.373	62.633	1.658	.371	.470	.622
12.000000	M135A	H249	56.908	57.533	.736	.170	.470	.130
16.000000	M109A	H155	57.282	57.058	-.219	.490	.570	.738
20.000000	A157	H146	-16.484	-16.042	.369	.848	.720	1.386

NO. OF REPEATED SERIES 0

F THE RATIO OF THE OBSERVED S.D. TO THE ACCEPTED S.D. IS LESS THAN THE CRITICAL F VALUE AND THEREFORE THE PROCESS IS TAKEN TO BE IN STATISTICAL CONTROL.

T THE T VALUE (THE RATIO OF THE DIFFERENCE BETWEEN THE OBSERVED VALUES AND THE ACCEPTED VALUES FOR THE CHECK STANDARD TO THEIR CORRESPONDING STANDARD DEVIATIONS) DOES NOT EXCEED THE CRITICAL VALUE OF 3. THEREFORE THE PROCESS IS REGARD AS BEING IN STATISTICAL CONTROL. THE STANDARD DEVIATIONS USED TO COMPUTE THE T VALUES WERE AS FOLLOWS: .540 .590 .630 .670 .760 .850 1.020

FIGURE 30. *Report of calibration.*

The results of measurements made at NBS on both long blocks, from 5 in to 20 in, and short blocks, 0.1 in to 4 in are presented in a "laboratory notebook" type of report. The report consists of three sections: an introduction which is reprinted in appendix 5; the statement of values and uncertainties which, for a typical long block set, is shown in figure 28; and an appendix which reports the state of the NBS measurement performance at the time the reported values were established, as shown in figures 29 and 30.

Referring to figure 28, the blocks for which the report applies are identified by owner and by serial number. The operator and the instrument used in making the comparisons are identified. The values, at 20 °C, are reported as block length, and as nominal block length and a correction. The uncertainty, which is plus or minus, is an expression of the limits within which values from repeated measurements are expected to fall. The systematic error component of the uncertainty relates to the uncertainty of the values assigned to the reference blocks used in the comparisons as previously discussed. The magnitude of the systematic error reflects the Process I ("new" interferometric) measurements made by NBS on the complement of reference standards. The random component of the uncertainty, 3 s.d., is based on the Process II (comparison process) performance parameters. The coefficient of expansion, in microinches per inch per °C, has been used to correct for small differences in temperature between the measurement environment and 20 °C. Since practically all long gage blocks are made from the same type of material, which is processed to obtain very nearly the same physical properties, no differential penetration corrections have been made.

This paper represents the efforts of many people over a span of several years. The cooperation and comments of Elmo Johnson and Dave Spangenberg of the Navy Eastern Standards Laboratory, and of J. C. Moody of Sandia Corporation, were most helpful. Geraldine Hailes, in addition to working with Joe Cameron on reference [6], prepared the initial computer programs for interfacing the measurement processes with the time-sharing computer. The statistical aspects of this paper are due primarily to Joe Cameron. Ruth Varner constructed programs to manage the very large amounts of data, and prepared the Report format. John Beers, Clyde Tucker, Grace Chaconas, Herb Badger, Ron Hartsock and Ruth Davenport were responsible for developing and operating the measurement processes as well as initially keeping track of all data. Horace Bowman's work on surface penetration of contacting probes was helpful. This work, still in progress, is essential for work with "short" blocks made from different materials. Those responsible for the execution were: Gertrude Tesler who patiently prepared the many typed drafts; Joanne Mobley who punched a very large number of data cards; and Hank Zoranski who prepared the art work. Finally, the comments of Karl Kessler, John Simpson and Jimmie Suddeth, who acted as "unofficial" readers, were invaluable.

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10. Appendix 1. Definition and Use of Length Value Assigned to a Block or Artifact

Definition

Artifacts, with two opposite faces essentially flat and parallel and generally in the form of rectangular parallelepipeds, are suitable length standards for a variety of uses. Ordered sets of such objects, available in several types and in lengths up to approximately 20 in, are usually called gage blocks. Following a concept of the perpendicular distance between a point and a plane as having a one-to-one correspondence with a characteristic common to many objects, one length of a gage block is the perpendicular distance between a definite gaging point on one surface of a block and a base plane in close proximity to the opposite surface, the distance being expressed in appropriate measurement units. This definition is used by the National Bureau of Standards and is also in general agreement with definitions used by other standards laboratories and organizations.

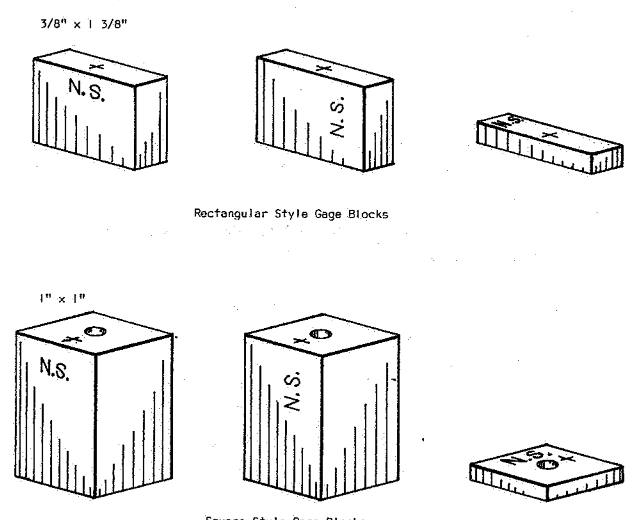
Specifying both the gaging point and the attitude of the block with reference to the base plane establishes a reasonably unique line interval to represent a "defined length."[19] The use of terminal points other than the specified gaging point, and variations in the method by which the base plane is brought into close proximity to the bottom of the block, may produce results which differ systematically from the length according to the definition. Failure to achieve a reasonable perpendicular between the defining line segment and the base plane, largely a matter of adjustment of the comparator or interferometer being used, may introduce small systematic errors (cosine errors). Variations in block geometry which affect the attitude of the block with reference to the base plane may introduce a variability in the measurement. The significance of variability from these sources must be judged relative to the precision of the measurement process in which the blocks are being used and relative to the functional requirements which are to be satisfied by the resulting measurement.

The dimensions of an artifact, one of which becomes the length by definition, are dependent on both the temperature of the artifact at the time of measurement and the historical age of the artifact. All materials respond to temperature changes by expanding or contracting in varying amounts. These changes are temporary and occur continuously. The relaxation and redistribution of stresses internal to the block change the dimensions of the block. These changes occur slowly and result in permanent changes in block dimensions. Careful selection of materials and control in the manufacturing process can reduce the magnitude of changes from these sources to some acceptable level. Regardless of the minimizing techniques, however, changes from both of these sources may be clearly observable in many precise measurement processes.

A measurement consists of performing a prescribed sequence of operations which include cleaning and establishing the attitude of the block with reference to the base plane as well as one or more intercomparisons with other blocks or with wavelength scales. The entire measurement effort from inspection to end result is called the measurement process. The result from the measurement process is an estimate of the length according to a particular definition and appropriate to the age of the gage block and its temperature at the time of the measurement. The practice used by the National Bureau of Standards to designate a "front," "top," "bottom," and a definite gaging point relative to the normal markings on a gage block is shown in figure 1 of this appendix.

Assuming that the thermal coefficientof expansion is reasonably linear in the neighborhood of 20 °C, and that the age dependent changes in dimensions can be adequately expressed by a linear function, an estimate of a defined length appropriate to any time and temperature can be predicted by the following relation:

$$L_m(t,T) = (N + \hat{Y}_m(t_o, T_o) + K_1(t - t_o))\,(1 + K_2(T - T_o) \pm (3\sigma_{\hat{Y}} + (N/R)S_R))$$



NOTES:

N.S. represents the nominal size marking on the block.
Front of the block is the side with the nominal size marking or the side to the right of the nominal size marking when it is marked on the gaging face.
Top of the block is the upper gaging face.
Bottom of the block is the lower gaging face.
Gaging Point is indicated by an X on the upper gaging face and is located at the center on rectangular blocks and is located midway between the hole and the front on hoke style blocks.

FIGURE 1. *"Gaging point" definition.*

[19] In this paper, length according to this definition is called the "defined length". It is also frequently called the "ISO" length.

where t_o, T_o refer to a specific time and temperature, and the subscript m, expressed in Roman numerals, designates the type of measurement used to establish $\hat{Y}$. Having initially established estimates of the parameters in this relation appropriate to a given block, further measurement efforts can be used to (a) make minor adjustments of the parameters or (b) verify the continued use of the relation to predict defined lengths for any time or temperature.

The right side of the above relation consists of two bracketed terms, the first establishes a numerical value in some set of consistent measurement units, and the second establishes an uncertainty for the numerical value. Considering each term in detail:

(1) $L_m(t, T)$ is the predicted value to be assigned as the defined length of a block at any time, t, and at any temperature, T. The subscript, m, expressed in Roman numerals, identifies the type of measurement process used to assign the basic numerical values. Where several types of measurements are involved, each must be clearly identified with an appropriate designator.

(2) N is an arbitrarily assigned numerical value exact to any required number of decimal places. The use of an arbitrary N reduces the magnitude of the numbers in some of the calculations, a convenience in hand computation but of little concern when data are processed by digital computers. N is usually chosen so that $|N-L| <$ the on-scale range of the available instrumentation.

(3) $\hat{Y}_m(t_o, T_o)$ is a numerical term which can be computed from current measurement data, or which can be established by a review of previous measurement data covering a long time span. $\hat{Y}_m(t_o, T_o)$ in combination with the arbitrary number N determines $L_m(t_o, T_o)$, the predicted length value assigned as the length of the defined interval at time, t_o, and temperature, T_o.

(4) K_1 is the first coefficient in the linear relation describing the dimensional changes of the gage block over a long time span. Since each block changes at a different rate, K_1 must be determined from a collection of Y's taken over a sufficiently long time span to establish the direction and amount of change for each block. If, relative to the precision of the measurement process, no long term change is taking place, $K_1 = 0$.

(5) $(t - t_o)$ is the time lapse, expressed in suitable units, since the establishment of an accepted $\hat{Y}_m(t_o, T_o)$.

(6) K_2 is the thermal coefficient of linear expansion of the gage block material in the direction of Y, or L. At the present time a handbook value for the material from which the gage block is constructed is normally used. Again, since each long block has a unique characteristic coefficient of expansion, it may be necessary to determine experimentally the appropriate value if the available process precision is to be utilized.

(7) $(T - T_o)$ is the expected, or actual, temperature difference between the gage block at the time for which the prediction is appropriate, and the temperature associated with the accepted $\hat{Y}_m(t_o, T_o)$.

The last terms in the relation are concerned with establishing the uncertainty with $\hat{Y}_m(t_o, T_o)$. The use of statistical methods to establish an uncertainty for the resulting value presumes that the measurement process is operating under some sort of reasonable statistical control. That is, in continuous operations, the results do not show grouping, bias or trends. As stated before, the measurement is the performance of a sequence of operations, some of which are comparisons, with the intent of establishing a quantitative value for the defined length of the block. Intercomparisons within a defined measurement permit the calculation of a standard deviation, σ_w, which is related to the measurement process.

Repeating a defined measurement procedure a number of times produces a sequence of numbers representing the characteristic of some object, in this case a sequence of Y's. One can compute another standard deviation, σ_t, for the collection of these results. One can also compute the standard deviation of the mean, or average $\sigma_{\hat{y}}$, which is representative of the confidence one can place on the average, or accepted, value for $\hat{Y}$. With this brief background, we can proceed with the description of the terms in the formula.

(1) The first term in the uncertainty brackets is the random component of the uncertainty. The computation of σ_y depends upon how $\hat{Y}_m(t_o, T_o)$ has been determined. For example, if the current estimate of $\hat{Y}_m(t_o, T_o)$ is the result of a repeated sequence of defined measurements over a relatively short time span, the formula would be:

$$\sigma_{\hat{y}} = \frac{k\sigma_w}{\sqrt{n}} \text{ or } \frac{\sigma_t}{\sqrt{n}}, \text{ whichever is larger.}$$

k is a factor that depends on the degree of redundancy in the defined measurement. σ_w is the within-group standard deviation, as described above, and n is the number of times the defined measurement has been repeated. If it is known that $\sigma_t > k\sigma_w$, as described above, then σ_t should be used. On the other hand, from a collection of $\hat{Y}_m(t_o, T_o)$ covering a long time span, one may want to determine a predicted value for some particular time by fitting a curve to the collection of points and using the extrapolated value for the time of interest as the best current estimate of $\hat{Y}$. In this case, the calculation of the random component of the uncertainty of $\hat{Y}$ is obviously a different formula.

(2) The last term is the systematic component of the uncertainty statement. This term is associated with the restraint on the defined measurement which permits number assignments to characteristics of unknown objects. The measurement procedures can only quantify differences, thus one or more of the objects must have assigned values, called restraints on the measurement process. S_R is the uncertainty of the numbers assigned to one or more objects used as restraints and is a measure of some prior measurement process performance. R is the total nominal length of the restraint blocks. The fraction N/R prorates the systematic error to the unknown blocks which are included in the current measurement. The manner in which the uncertainty is treated as one moves from one laboratory to another is explained in figure 2, an excerpt from NBS Monograph 103.

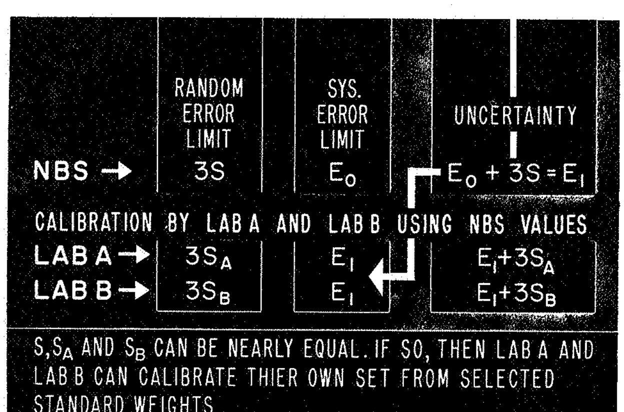


FIGURE 2. *Uncertainty in a calibration sequence (excerpt from Monograph 103).*

Use

Stating the defined length of a block or artifact in terms of $(N+\hat{Y})$ suggests two different interpretations. Since N is exact (the nominal length), $(N+\hat{Y})$ carries the uncertainty of $\hat{Y}$, thus when disseminating a length unit, one is concerned with the uncertainty of $\hat{Y}$. On the other hand, in many instances the interest is in the $|\hat{Y}|$ relative to some particular requirement. That is, if $|\hat{Y}|$ is less than some limiting value, the block is used as if the length was N. Unfortunately, these two methods of interpretation are not well understood. In the first interpretation, the uncertainty of $\hat{Y}$ reflects all of the terms in the above relation. In the second interpretation, compliance with specification limits is usually announced on the basis of a simple uncharacterized measurement procedure.

There are several courses of action dependent upon the intended usage. When the uncertainty of $\hat{Y}$ is smaller than the tolerance limits, one can accept the $\hat{Y}$ and its uncertainty in lieu of the specified limits. For example, a length 4.000 028 ± .000 002 in as determined by measurement is a more precise basis for adjusting instruments, etc., than a statement that the length of the block does not deviate from a nominal 4 in excess of 0.000 005 in. Such action, however, carries the implication that all of the terms considered in establishing the uncertainty of $\hat{Y}$ must also be considered in the local measurement process in which the block is to be used.

In certain circumstances, one can use simplified procedures to establish tolerance compliance. If the measurement process used is free from significant systematic errors (the magnitude of known systematic effects is less than one s.d.) and if the process standard deviation is less than approximately one-tenth the tolerance limit, a simple sorting procedure should identify blocks which are significantly "out of tolerance." Reasonable tolerance limits should encompass the combined uncertainty of the production and inspection measurement process.

Finally, one can evaluate the situation relative to a particular end use and accept those items which are adequate. Generally speaking, one cannot compare the results for the same measurement performed by two different processes unless both processes are well characterized. One cannot judge the difference between the results without a detailed knowledge of both processes and the methods of computation (round off rules, etc.). This is particularly true when one measurement is in essence a sorting operation according to a locally determined procedure. In many instances, the use of precise measurement processes to establish an "in tolerance by actuality" will not confirm an "in tolerance by local definition."

11. Appendix 2. Gage Block Intercomparison Designs

With the sequence of operations required to make a "single measurement" precisely defined, the schedule of "single measurements" to be made between one or more "knowns" and a group of "unknowns" is called an intercomparison design. In general, the intercomparison design provides a means to obtain the most information from the fewest measurements. While many features can be incorporated, the formulation of efficient designs is not a trivial task. Discussion of design formulation is beyond the scope of this paper.

The sequence of operations required for a "single measurement" can be shown symbolically as:

$$Y_1 = \mathscr{X}_1 - \mathscr{X}_2 + \text{random error}$$

where $\mathscr{X}_1, \mathscr{X}_2$ are the unknown magnitudes of the property of interest embodied in each of two objects; and Y_1 is the observed difference in magnitude expressed in appropriate measurement unit. A design which requires difference measurements between all pairs in group of four objects would require the following measurements:

Property	Observations
$\mathscr{X}_1 - \mathscr{X}_2$	Y_1
$\mathscr{X}_1 - \mathscr{X}_3$	Y_2
$\mathscr{X}_1 - \mathscr{X}_4$	Y_3
$\mathscr{X}_2 - \mathscr{X}_3$	Y_4
$\mathscr{X}_2 - \mathscr{X}_4$	Y_5
$\mathscr{X}_3 - \mathscr{X}_4$	Y_6

In matrix notation, this group of equations can be expressed as: + random errors

$$\begin{bmatrix} 1 & -1 & 0 & 0 \\ 1 & 0 & -1 & 0 \\ 1 & 0 & 0 & -1 \\ 0 & 1 & -1 & 0 \\ 0 & 1 & 0 & -1 \\ 0 & 0 & 1 & -1 \end{bmatrix} \begin{bmatrix} \mathscr{X}_2 \\ \mathscr{X}_1 \\ \mathscr{X}_3 \\ \mathscr{X}_4 \end{bmatrix} = \begin{bmatrix} Y_1 \\ Y_2 \\ Y_3 \\ Y_4 \\ Y_5 \\ Y_6 \end{bmatrix}$$

or

$$A\mathscr{X} = Y$$

The solution for $\mathscr{X}$ by the method of least squares is:

$$\mathscr{X} = (A'A)^{-1}A'Y$$

However, since only the differences have been measured, $(A'A)$ is singular. One or more of the objects being compared can be grouped as a restraint and the sum of the values, R, used to represent the magnitude of the property of interest embodied in each of the objects, relative to the be used to augment the above relation. The estimate of the magnitude of the property of interest embodied in each of the objects, relative to the magnitude assigned to the "known" objects or objects, that is, the restraint, becomes:

$$\hat{\mathscr{X}} = \begin{bmatrix} A'A & \mathscr{r} \\ \mathscr{r}' & O \end{bmatrix}^{-1} \begin{bmatrix} A'Y \\ R \end{bmatrix}$$

where $\mathscr{r}' = (\mathscr{r}_1, \mathscr{r}_2 \, . \, . \, . \, \mathscr{r}_k)$ is a vector of the coefficients in the restraint

$$\mathscr{r}_1\mathscr{X}_1 + \mathscr{r}_2\mathscr{X}_2 + \, . \, . \, . \, . \, \mathscr{r}_h\mathscr{X}_k = R.$$

Note that in the above, the script letters refer to the magnitude of the property acting on the comparison instrument, and the italic letters are the numbers assigned as estimates of the magnitude of the property relative to a particular restraint value R.

The standard deviation of the group of comparisons can be obtained by defining Δ to be the difference between the observed value, Y, and the expected value based on the estimated values:

$$s_w = \sqrt{\frac{\Sigma\Delta^2}{n-k+1}}$$

for a design with n observations on k objects.

Since all of the comparisons required by a given design can usually be made on one instrument in a short time interval, the standard deviation computed from the residual from one sequence of comparisons is called the estimated within group precision, s_w, for a particular instrument. This standard deviation applies to the defined "single measurement." For a given instrument, each defined "single measurement" procedure will have a distinctive standard deviation. Collections of s_w can be combined to obtain a long term or accepted within group standard deviation, σ_w, one of the important process performance parameters.

The flexibility of intercomparison designs provides a means for the metrologist to obtain long sequences of repeated measurements on the same objects with little additional measurement effort. If one is to believe that the values assigned to the "unknowns" are valid over time, the fact must be demonstrated. The idea of a "check standard" refers to a difference between two objects, or the value assignment to an object, the objects being similar in all respects to the "unknown" and always used in a particular measurement. For example, in the design shown, an object with known value could be designated $\mathscr{X}_1$. This object would be called the "starting standard" since its assigned value, $\mathscr{X}_1$, would be the restraint, $\mathscr{X}_2$ could be the "check standard," assumed unknown and always used with $\mathscr{X}_1$. The sequence of measurements called for by the design would assign values to $\mathscr{X}_2$, $\mathscr{X}_3$, and $\mathscr{X}_4$ relative to $\mathscr{X}_1$. While the objects $\mathscr{X}_3$ and $\mathscr{X}_4$ together with their assigned values are passed on to others, $\mathscr{X}_2$ remains with the process. The collection of values for $\mathscr{X}_2$ reflects not only the variation of both $\mathscr{X}_1$ and $\mathscr{X}_2$ but also the variability of the process over time. The standard deviation of this collection of values is called the "total standard deviation of the process," σ_T.

The appropriate choice of location within the design for the "starting standard(s)" and the "check standard" is part of the design formulation. Where possible, for the type of design shown, both $\mathscr{X}_1$ and $\mathscr{X}_2$ are used for "starting standards." The restraint is taken as the sum of the assigned values, $(\mathscr{X}_1+\mathscr{X}_2)$ and the difference between $\mathscr{X}_1$ and $\mathscr{X}_2$ as determined from the measurements serves as a "check standard." This procedure can sometimes reduce the systematic component of the uncertainty of the values assigned to the unknowns. (The systematic error of the restraint is prorated between the unknowns in proportion to the ratio of the value of the unknown to the value of the restraint. This will be discussed in detail elsewhere.)

The design shown is usually called a "four one's" design, four being the number of objects involved and the one being associated with the limitations of the "on scale" range of the various measurement instruments. For the most part, available precise instruments have a limited on scale range so that the objects being compared must be nominally equal. It should be noted that this is not a limitation imposed by the statistical design.

The normal procedure for describing a design is to show the A matrix, in terms of + and − signs (omitting the ones and zeros). The columns are labeled with the nominal values of the objects being intercompared, and the rows are labeled with an identification for the results of the prescribed comparison. The restraint vector is shown, and in some cases, the location of the "check standard." The design previously described could be shown as:

	1 − 1	1 − 2	1 − 3	1 − 4
$Y(1)$	+	−		
$Y(2)$	+		−	
$Y(3)$	+			−
$Y(4)$		+	−	
$Y(5)$		+		−
$Y(6)$			+	−
R	+	+		
C	+	−		

This would be interpreted as meaning the difference, as measured by the prescribed procedure, between object 1–1 and object 1–2 is called $A(1)$, and so on. The restraint, R, is the sum of the values currently assigned to 1–1 and 1–2. The check standard, C, is the difference between the values determined in the process for 1–1 and 1–2. The position of the restraint would be shown in vector form, $R(1,1,0,0)$, and the check standard location would be shown as $C(1,-1,0,0)$. If only the first object, 1–1 had an assigned value, the restraint vector would be $R(1,0,0,0)$, and the check standard would most likely be the second object, 1–2, designated by the vector $C(0,1,0,0)$. For a fuller treatment of this subject see reference [6].

12. Appendix 3. Gage Block Interferometry

A typical gage block interferometer is shown schematically in figure 1. A beam of collimated monochromatic light impinges on a beam splitter part of which passes through to reference mirror M1, and part of which is reflected to the platen or reference mirror P1. The reflected beam from P1 passes through the beam splitter into the viewing system. For the purpose of this discussion, the reflected light from M1 can be thought of as coming from the Virtual Image M1, hence also passing through the beam splitter into the viewing system. The beams are recombined in the viewing system to produce the observed interference fringe patterns. It should be noted that in such a schematic diagram, all angles must be shown very large. In the real

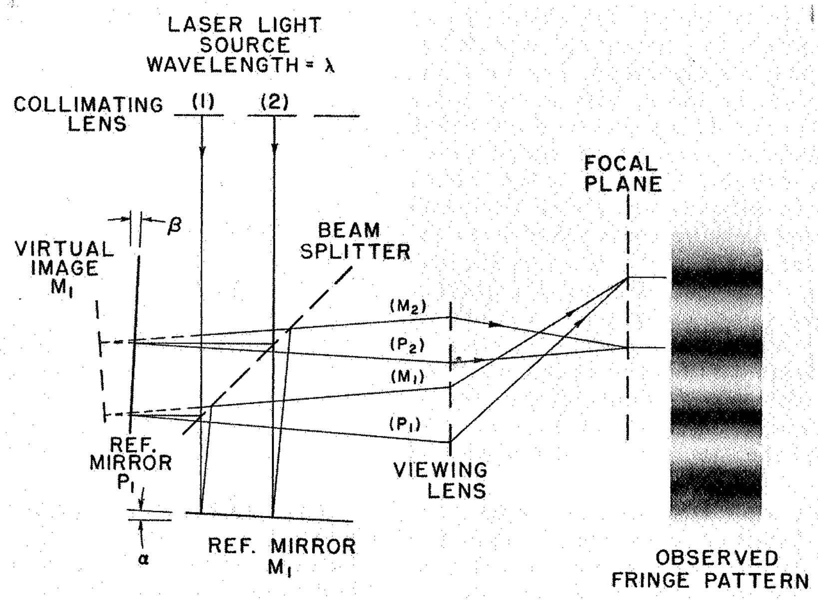


FIGURE 1. *Schematic gage block interferometer.*

instrument, all angles are very small so that cosine length errors are practically negligible.

In detail, an arbitrary ray (1) divides at the beam splitter, one part being reflected from P1 along path (P1) to the focal plane, and one part being reflected from Virtual M1 along path (M1). The difference in path length, starting at the beam splitter and ending at the focal plane is of interest. For some position across the face of the platen, this difference in path length will be an odd multiple of the half wavelength of the light. Under this condition, the two ray components will interfere destructively at the focal plane, and at that point in the observed field would be the dark center of an interference fringe. Because of the included angle $(\alpha + \beta)$, between P1 and Virtual M1, the difference in path length for the two ray incident components continually changes as one moves across the viewing field. For ray (2) the difference will again be an odd multiple of the half wavelength, indicating the center of the second fringe. Midway between ray (1) and ray (2), the difference in path length is an even multiple of the half wavelength, therefore in this region there is no destructive interference, thus the color of the light is seen. In the field of view the resulting fringe pattern appears as alternate rows of dark and colored bands.

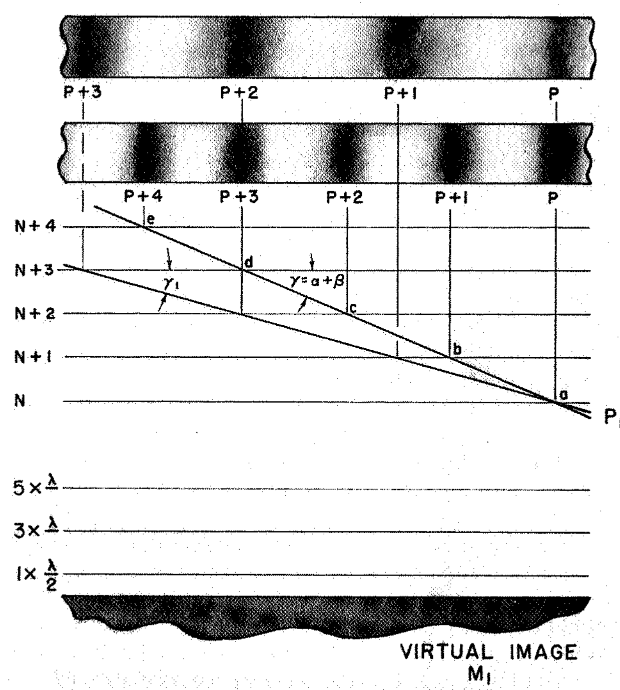


FIGURE 2. *Adjusting interference fringe pattern.*

The fringe pattern can be interpreted as shown in figure 2. Starting with Virtual M1, one can construct a series of parallel planes representing the difference in path length in odd multiples of the half wavelength. Except for the first plane, these planes represent incremental changes in elevation above Virtual M1 of one wavelength. The intersection of surface P1 with these elevation planes, at points a through e, designates the centers of the observed interference fringes when the angle of intersection is $(\alpha+\beta)$. While the order of the observed fringes is not known, starting at point a, the center of Fringe P, point b, the center of fringe $P+1$, is one wavelength higher in elevation, and so on. If the intersection angle, γ, is decreased by changing either α or β, or both, the fringes appear to broaden and spread out, as shown in the top part of the figure 2. If the operation is done slowly, one can observe fringe $P+1$ move to a new location. Fringe $P+4$ would move completely out of view.

With a gage block on the platen, as shown in figure 3, the top surface of the block intersects another set of parallel elevation planes in a similar manner. If the block length L, was shortened by the amount dL, as shown, fringe $B+1$ would be coincident with fringe $P+2$. In like manner, if L was increased an appropriate amount, fringe $B+1$ would become coincident with fringe $P+3$. From this, it follows that the difference in optical path length associated with fringes $B+1$ and $P+2$ is:

$$((B+1)-(P+2)+(a/b))\lambda$$

where $(B+1)-(P+2)$ is a large integer (Int. F), which must be determined by other means and (a/b) is the observed fractional fringe. This path length difference is equivalent to $2L$, so that:

$$L=(\text{Int.}\,F+(a/b))\;(\lambda/2).$$

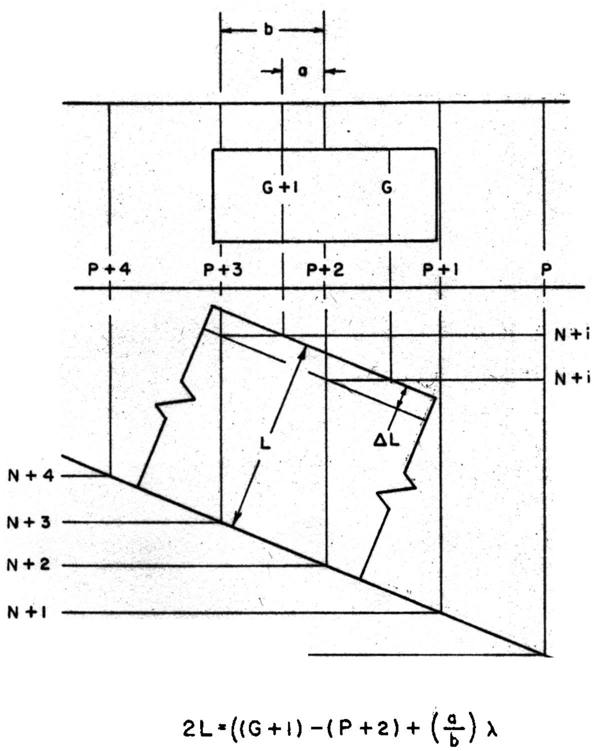


FIGURE 3. *Block length in terms of fringe order.*

In practice, the angle of inclination of $P1$ with respect to M1 is adjusted to obtain several fringes across the top of the block, with one fringe centered very nearly over the defined gaging point. Differences, if any, in the optical properties of the block and the platen have the effect of changing the path length differences in a manner not related to L. For long blocks, small platens are used which are of similar material and surface finish as the blocks in order to obtain nearly the same optical properties on both surfaces. For a given setup, one must determine experimentally the direction of increasing fringe order.

For a particular measurement, a "tentative" assigned length, $L(t, T)$ is expressed in "fringe" by:

$$F=2(L(t,T))/\lambda_{T,p,f}=(\text{Int.}\,F+\Theta)$$

where $\lambda_{T,p,f}$ is the wavelength of the laser radiation at the time of the measurement; T, p and f being the air temperature, pressure and relative humidity at the time the fringe photograph is taken, and Θ is the computed fraction. In practice for well known reference blocks such as the NBS (.) blocks considered in section 5.0, the accepted value, $L_1(t, 20)$, is normalized to temperature T for $L(t, T)$. For other blocks, such as the NBS(. .) group considered in section 6.2, $L(t, 20)$ is determined by mechanical comparison with suitable reference blocks.

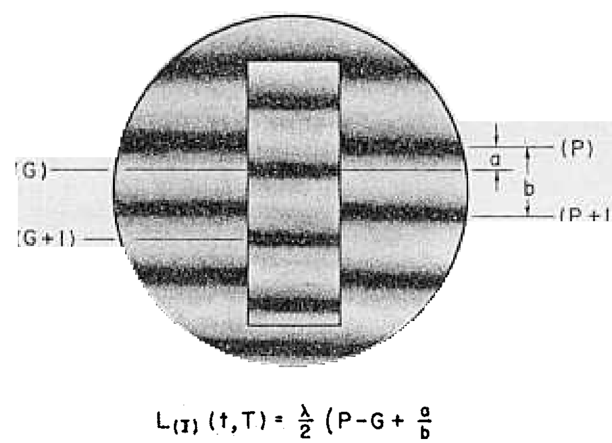


FIGURE 4. *Observed fringe pattern and interpretation.*

To interpret the fringe photograph, figure 4, fringes of increasing order P, $P+1$, etc. are associated with the platen, and fringes of order B, $B+1$, etc. are associated with the gaging surface of the block. Thus:

$$F=(P\quad B+a/b)$$

where the ratio, a/b, is the "observed" fringe fraction, Θ_0. Generally Θ_0 is simply substituted for the fractional part of F so that:

$$F'=(\text{Int.}F)+\Theta_0$$

and

$$L_I(t,\, T) = F'\,(\lambda_{T,p,f})\left(\frac{1}{2}\right)+s$$

where s, the interferometer aperture correction, is added.

There are cases in which the last digit in (Int. F) must be raised or lowered by one. For example, if the fractional part of F is 0.98 and Θ_0 is 0.03, obviously the last whole number in F must be raised by one before adding Θ_0. Finally, the value is normalized to $T=20°$ C:

$$L_I(t,20)=L_I(t,T)\,(1+K_2(20 \quad T))$$

It should be noted that the above relation determines a total length value, not a deviation from a nominal value. When the assigned value is expressed in corrections to a nominal length, N, the correction $Y_1(t, 20°)$ is computed as follows:

$$L_I\,(t,20)=N+Y_1\,(t,20)\,.$$

With the availability of "fringe counting" interferometers, the task of establishing an initial estimate of the length of a block suitable for use in single wavelength interferometry is greatly simplified. Such an instrument, shown schematically in figure 5, uses a divergent light source. As before, the central ray impinges on the beam splitter, with one component being reflected from mirror M1. In the position shown, the path difference between the two components is an odd multiple of the half wavelength, so that the observed pattern reflects destructive interference. The components of the divergent rays R2 follow longer path lengths (R2) and (M2), which again differ by an odd multiple of the half wavelength. The result is a "bull's eye" pattern. As the moving mirror M1 moves by the amount dL, with the path (R1) fixed, the difference in path length for the central ray components relates dL directly to the half wavelength of the light source. If M1 moved by the amount dL is equivalent to a path length change of one half-wavelength, the conditions for destructive interference do not exist for the central ray, and the center fringe disappears. Adding additional movement of the amount dL will again cause the center fringe to appear. A light sensitive detector focused on the center of the observed pattern will not only "count" the fringes as mirror M1 is moved, but also will estimate the fractional fringe change. While this instrument is in essence making a displacement measurement, when coupled with a suitable surface detector and mounted in a suitable frame, estimates of length can be established by a procedure such as illustrated in figure 6.

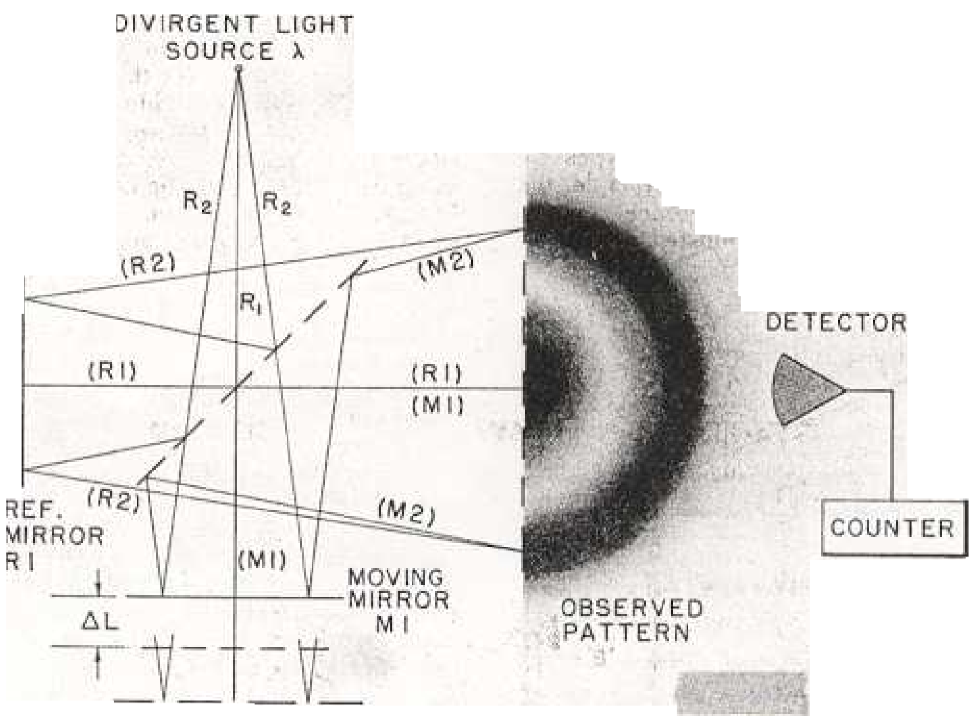


FIGURE 5. *Schematic "fringe counting" interferometer.*

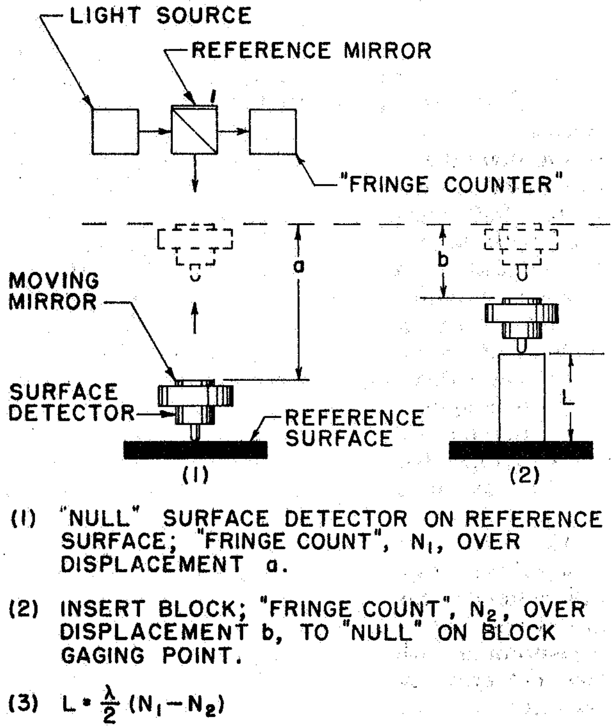


FIGURE 6. *Arrangement to determine integral fringe order.*

13. Appendix 4. The Gage Block Comparator

Generally gage block comparators which utilize contacting probes to detect the block surface are called mechanical comparators. Several types are shown schematically in figure 1. In principle, the separation, s, between two reference planes, "A" – "A" and "B" – "B", is adjusted so that for some y, an "on scale" condition exists for both objects to be compared. At the microinch level, the "on scale" range is usually limited so that the two objects being compared must be very nearly identical in size. Instruments differ in the way the reference planes are defined, and in the way in which the movement of the contacting probe, that is a change in y, is detected and quantized.

For instruments of the type illustrated by figure 1, (1), the bottom reference plane, "B" – "B", is the interface between the surface of comparator anvil and the bottom gaging face of the block. The anvil surface must be reasonably flat with a surface finish such that the block will not "wring" to the anvil. The top reference plane is established by some "zero" electrical plane associated with the top transducer. In most cases, the transducer is a sophisticated linear variable differential transformer (LVDT). Displacement of the moving element from the electrical "zero" produces a signal which is proportional to the change in y. Scale shift such as adjusting to obtain a particular instrument indication for a given block, and scale span (microinches per reading scale division), can be accomplished by adjusting the electrical circuitry. The tip of the probe is often a diamond ground to a spherical shape with a particular radius. The force, F, acting on the probe under contacting conditions can be adjusted. In use, both the contact pressure and the span should be checked periodically.

In use, block 1, standing vertically on plane "B" – "B", is moved into the measuring position by sliding it gently under the probe until the point of contact is very nearly identical with the defined gaging point, such as shown in figure 2. The coordinate $y(1)$, from the reference plane to the interface between the probe and the block surface, relates to the instrument indication. Noting the indication, 0_1, block 1 is removed and block 2 inserted in

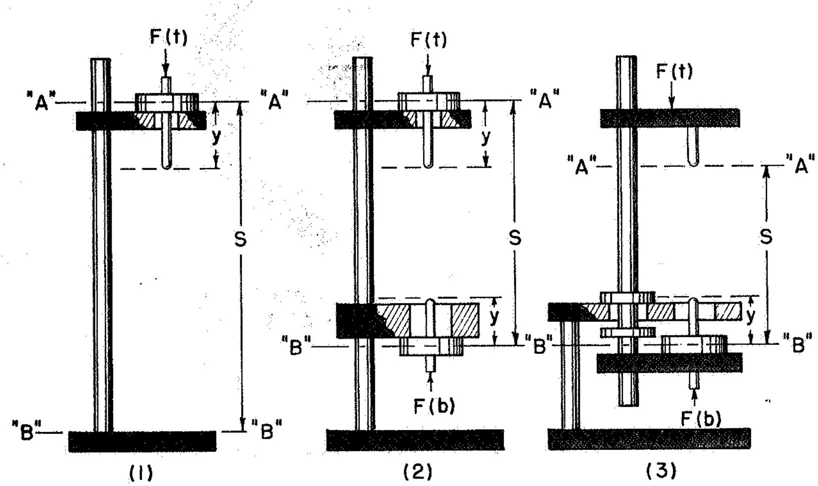
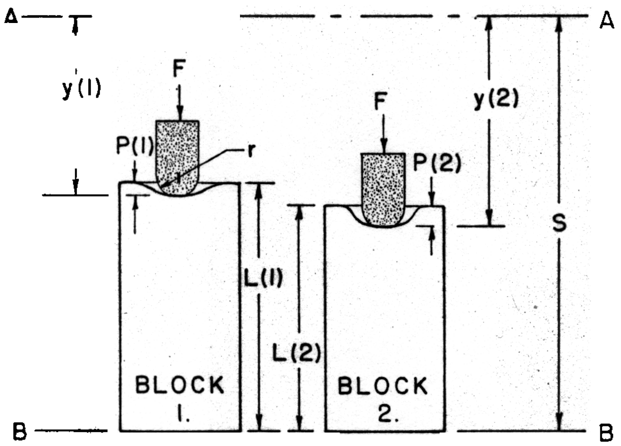


FIGURE 1. *Schematic gage block comparator.*



$$S = L(1) - P(1) + y(1) = L(2) - P(2) + y(2)$$

$$L(1) - L(2) = y(2) - y(1) + P(1) - P(2)$$

$$= y(2) - y(1) + \beta$$

but

$$y(1) = -K(O_1 + h)$$

$$y(2) = -K(O_2 + h)$$

so that

$$L(1) - L(2) = K(O(1) - O(2)) + \beta + \epsilon$$

FIGURE 2. *Differential penetration, β.*

the same manner. With 0_2, the indication obtained for block 2, the difference in length can be determined from the relation:

$$L(1) - L(2) = K(0_1 - 0_2) + \beta + \epsilon$$

where K relates the instrument observations to measurement units, (microinches per reading scale division); β accounts for the difference in penetration, ($P(1) - P(2)$), and ϵ accounts for the error of measurement. In most cases, the instrument is adjusted so that $K = 1$.

The penetration, P, is a function of the force on the probe, the radius of the tip and the physical properties of the tip, as well as the surface and physical characteristics of the gaging face of the block. For a given probe and contact force, as long as the characteristics of the blocks being compared are nearly the same, β is essentially zero. If the characteristics of the blocks is such that β is large relative to the precision of the comparator, two courses of action can be taken. One can adjust F for each block in order to maintain $\beta = 0$, or one can correct the observed data to account for $\beta \neq 0$. For most commercial comparators, the latter course of action must be taken. In both cases, the magnitude of the force, or the magnitude of the correction, must be determined by independent experiments.

For instruments of the type shown in figure 1(b), two contacting probes and two transducers are used. In this case the reference planes are the electrical "zeros" of the two transducers. The bottom anvil is merely a support plane to hold the block in a reasonably reproducible attitude at the time of measurement. Such instruments are used in the same manner as the instruments of type 1(a). Normally one can "read" the individual outputs of both transducers, or the difference between the outputs. When blocks of different materials are being compared, β must be determined for both the top and bottom contact probes.

In the arrangement shown in figure 1(c), the fixed top contact established a "point" reference in the top reference plane which is through the interface between the tip of the probe and the top of the block. The bottom reference plane is again the electrical "zero" of the transducer. In use, the yoke is raised to permit inserting the block into the measuring position. The yoke is lowered into the reading position shown. The top contact pressure is adjusted by means of springs and counterweights acting on the yoke, and the bottom contact pressure is adjusted at the transducer. Again, if the blocks being compared have different physical properties, β must be determined for both contacts.

Appendix 5.

U. S. DEPARTMENT OF COMMERCE
NATIONAL BUREAU OF STANDARDS
INSTITUTE OF BASIC STANDARDS
WASHINGTON, D.C. 20234

R E P O R T

O F

L E N G T H   V A L U E S

DIMENSIONAL TECHNOLOGY
NATIONAL BUREAU OF STANDARDS
WASHINGTON D.C. 20234
DOALL
GRADE AA
APRIL 02-04 1974

TEST NUMBER 210448

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TECHNOLOGY SECTION

INTRODUCTION

THIS DOCUMENT IS A COMPREHENSIVE REPORT COVERING THE SEQUENCE OF OPERATIONS USED TO ASSIGN LENGTH VALUES TO THE ARTIFACTS IDENTIFIED ABOVE. IT INCLUDES A DESCRIPTION OF THE MEASUREMENT METHODS AND PROCEDURES WHICH WERE USED, AND A SUMMARY OF THE ANALYSIS OF THE MEASUREMENT DATA. THE ARTIFACTS HAVE BEEN DIVIDED INTO GROUPS AS FOLLOWS:

GROUP I	LESS THAN		.1	IN
GROUP II	.1	TO	.107	IN
GROUP III	.108	TO	.126	IN
GROUP IV	.127	TO	.164	IN
GROUP V	.147	TO	.500	IN
GROUP VI	.55	TO	4.0	IN
GROUP VII	5.0	TO	20.0	IN

THE ASSIGNED LENGTH VALUES, THE THERMAL COEFFICIENTS OF EXPANSION (ASSUMED OR MEASURED AS NOTED) AND THE UNCERTAINTIES OF THE VALUES ARE PRESENTED. IN THE APPENDIX THE STATISTICAL INFORMATION SHOWN BECOMES A PART OF THE COLLECTION OF DATA USED TO CHARACTERIZE THE NBS MEASUREMENT PROCESS. SUCH A COLLECTION HAS BEEN USED TO ESTABLISH THE CONTROL LIMITS FOR SURVEILLANCE OF THE MEASUREMENT PROCESS AND TO GIVE ASSURANCE OF VALIDITY OF STATEMENTS ABOUT THESE MEASUREMENTS. THESE COLLECTIONS ARE OPEN FOR INSPECTION AT OUR FACILITY. IT IS PRESUMED THAT THESE ARTIFACTS WILL BE USED IN A SIMILARLY WELL-CHARACTERIZED MEASUREMENT PROCESS SO THAT THE STATISTICAL PARAMETERS OF BOTH PROCESSES CAN BE COMBINED TO PROVIDE A REALISTIC ESTIMATE OF THE UNCERTAINTY OF THE LENGTH UNIT AS ACTUALLY REALIZED IN ANOTHER FACILITY. A COMPREHENSIVE SERVICE DIRECTED TOWARD SUCH AN EVALUATION IS PART OF A LENGTH MEASUREMENT ASSURANCE PROGRAM OF THE NATIONAL BUREAU OF STANDARDS.

LENGTH MEASUREMENT

THE ARTIFACTS COVERED BY THIS REPORT WERE CLEANED AND TREATED (LIGHTLY 'STONED') TO REMOVE SURFACE IMPERFECTIONS WHICH MIGHT INTERFERE WITH THE MEASUREMENT. ALL, OR SAMPLES, HAD BEEN TESTED FOR THE ABILITY TO ADHERE CLOSELY ('WRING') TO SUITABLE FLAT SURFACES AND TO EACH OTHER. NO TESTS HAVE BEEN MADE TO ASCERTAIN THE DEGREE OF 'FLATNESS' OF THE 'GAGING' SURFACES OR THE DEGREE OF 'PARALLELISM' OF THE TWO 'GAGING' SURFACES. SINCE NO 'GAGING' SURFACES ARE EITHER FLAT OR PARALLEL, IT IS FELT THAT THE MOST IMPORTANT ATTRIBUTE OF THE ARTIFACT IS THE ABILITY TO BE MADE TO ADHERE CLOSELY TO APPROPRIATE SURFACES (I.E., WRING). WHERE IT IS FELT THAT QUANTITATIVE ESTIMATES OF THE DEGREE OF 'FLATNESS AND PARALLELISM' ARE REQUIRED, IT IS SUGGESTED THAT ACCEPTED TESTS BE PERFORMED AT THE POINT OF USAGE.

THE LENGTH VALUES ASSIGNED TO THE ARTIFACTS IN THIS REPORT ARE WITH REFERENCE TO THE VALUES ASSIGNED TO SELECTED ARTIFACTS OF NBS. THE REFERENCE VALUES HAVE BEEN ESTABLISHED BY AN INTERFEROMETRIC

MEASUREMENT PROCESS AND ARE THE LENGTHS OF A LINE FROM A DEFINED 'GAGING' POINT 'X' ON ONE SURFACE TO AN AUXILIARY PLANE IN CLOSE PROXIMITY TO THE OPPOSITE SURFACE. FOR LOCATION OF THE POINT, X, RELATIVE TO THE NOMINAL SIZE MARKING DENOTED BY 'SIZE' SEE THE ACCOMPANYING FIGURE.

INTERCOMPARISON DESIGN

GROUPS OF FOUR ARTIFACTS OF THE SAME NOMINAL SIZE, TWO REFERENCE AND TWO 'UNKNOWN,' ARE INTERCOMPARED ACCORDING TO THE FOLLOWING DESIGN:

OBSERVATION	DIFFERENCE MEASURED
Y(1)	S. - S..
Y(2)	Y - S.
Y(3)	X - Y
Y(4)	S.. - X
Y(5)	S.. - Y
Y(6)	Y - S.
Y(7)	S. - X
Y(8)	X - S..

THE SYMBOLS (.) AND (..) INDICATE REFERENCE ARTIFACTS, LISTED BY SERIAL NUMBER IN THE BODY OF THE REPORT. (X) AND (Y) DESIGNATE 'UNKNOWN' ARTIFACTS, ONE SET OF WHICH IS COVERED BY THIS REPORT, AND THE OTHER COVERED BY TEST NUMBERS STATED IN THE APPENDIX TO THE REPORT.

IN SUCH AN INTERCOMPARISON, ONLY DIFFERENCES IN LENGTH CAN BE MEASURED. BECAUSE OF THE LIMITED RANGE OF PRESENT COMPARATORS, ALL ARTIFACTS IN A GIVEN COMPARISON ARE OF THE SAME NOMINAL VALUE. A REDUNDANCY IN THE NUMBER OF MEASUREMENTS (EIGHT MEASUREMENTS TO DETERMINE FOUR VALUES) PROVIDES A MEANS FOR CHECKING ON THE PRECISION OF THE PROCESS BY THE METHOD OF LEAST SQUARES USING THE SUM OF THE LENGTHS OF THE TWO REFERENCE ARTIFACTS, ((.) + (..)), AS THE RESTRAINT. THE COMPUTED DIFFERENCE BETWEEN THE REFERENCE ARTIFACTS ((.) - (..)) IS AN INDEPENDENT ESTIMATE OF THE DIFFERENCE, AND SERVES AS A 'CHECK STANDARD'.

PROCESS CONTROL

THE STANDARD DEVIATION, AS COMPUTED FROM THE LEAST SQUARES SOLUTION, PROVIDES A CHECK ON THE SHORT TERM, OR 'WITHIN-RUN' PROCESS PRECISION. THIS VALUE IS COMPARED WITH THE LONG RUN AVERAGE OF THESE STANDARD DEVIATIONS DESIGNATED THE ACCEPTED WITHIN-RUN STANDARD DEVIATION OF THE PROCESS FOR THE GROUP.

THE VALUES OBTAINED FOR THE DIFFERENCE IN LENGTH BETWEEN THE TWO 'KNOWN' REFERENCE ARTIFACTS PROVIDE, AS TIME GOES ON, A SEQUENCE OF VALUES THAT REALISTICALLY REFLECT THE TOTALITY OF VARIATIONS WHICH BESET MEASUREMENTS OF TEST ITEMS. THE STANDARD DEVIATION OF THIS COLLECTION OF VALUES IS THE TOTAL

PROCESS STANDARD DEVIATION.

IF THE 'WITHIN-RUN' STANDARD DEVIATION AND THE VALUES FOR THE CHECK STANDARD CAN BE REGARDED AS MEASUREMENTS FROM STABLE PROBABILITY DISTRIBUTIONS AND THE TESTS OF THE VALUES FROM THE CURRENT RUN CONFORM TO THEIR RESPECTIVE DISTRIBUTIONS THEN ONE TAKES THIS AS EVIDENCE THAT THE PROCESS IS IN CONTROL, AND THAT PREDICTIVE STATEMENTS REGARDING UNCERTAINTY ARE VALID.

SYSTEMATIC ERRORS IN USE OF BLOCK

IN THE USE OF THESE BLOCKS IN PRACTICAL MEASUREMENT, TWO FACTORS MAY INTRODUCE SYSTEMATIC ERRORS INTO THE RESULTS: DISSIMILARITY OF MATERIAL AND DEVIATION OF TEMPERATURE FROM 20 C.

LENGTH VALUES CAN BE ASSIGNED TO OTHER LIKE BLOCKS BY DETERMINING THE DIFFERENCE IN LENGTH WITH A CONTACTING COMPARATOR. IF THE BLOCKS ARE NOT SIMILAR, THE INDICATED DIFFERENCE IS A FUNCTION OF THE FORCE EXERTED BY THE COMPARATOR PROBE ON THE ARTIFACTS UNDER COMPARISON AS WELL AS THE ELASTIC PROPERTIES AND SURFACE GEOMETRY OF BOTH THE PROBE AND ARTIFACTS IN THE IMMEDIATE VICINITY OF THE POINT OF CONTACT. DATA ADJUSTMENT TO COMPENSATE FOR THESE DIFFERENCES MAY BE NECESSARY. IF THE COMPARATOR BEING USED HAS BOTH A TOP AND BOTTOM CONTACT, THE DIFFERENTIAL PENETRATION FOR BOTH CONTACTS MUST BE CONSIDERED. THE UNCERTAINTY FOR THE REPORTED VALUES SHOULD INCLUDE AN ALLOWANCE FOR THE UNCERTAINTY OF THE DIFFERENTIAL PENETRATION CORRECTIONS. WHEN THE TWO ARTIFACTS ARE MADE OF THE SAME MATERIAL AND TESTED BY THE SAME PROBE(S) AT THE SAME CONTACT FORCE, NO CORRECTION IS APPLIED.

ALL MEASUREMENTS FOR THIS REPORT WERE MADE IN A TEMPERATURE ENVIRONMENT IN THE NEIGHBORHOOD OF 20 C. ASSIGNED VALUES HAVE BEEN ADJUSTED TO THAT APPROPRIATE FOR USE IN AN ENVIRONMENT OF 20 C (1968 INTERNATIONAL PRACTICAL TEMPERATURE SCALE) USING THE STATED, OR HANDBOOK,VALUES FOR THE TEMPERATURE COEFFICIENT OF LINEAR EXPANSION. IN THE COMPARISON PROCESS, ALL ARTIFACTS ARE AT VERY NEARLY THE SAME TEMPERATURE. CORRECTIONS BASED ON DIFFERENTIAL COEFFICIENTS OF EXPANSION ARE ASSUMED NEGLIGIBLE FOR ARTIFACTS OF GROUPS I THROUGH VI. IN ORDER TO EXTEND THE USEFULNESS OF THE ASSIGNED VALUES OVER A TEMPERATURE RANGE OF 20 C TO 25 C, IT MAY BE NECESSARY TO DETERMINE A COEFFICIENT OF EXPANSION FOR EACH ARTIFACT OF GROUPS VI AND VII. A PROCEDURE TO DO THIS IN THE NORMAL COURSE OF MEASUREMENT IS NOW UNDER DEVELOPMENT. MEASURED COEFFICIENTS OF EXPANSION WILL BE ASSIGNED TO EACH ARTIFACT WHEN AVAILABLE.

UNCERTAINTY

THE PREDICTED, OR ACCEPTED, VALUES OF THE REFERENCE ARTIFACTS ARE ESTIMATES OF THE LENGTH AT 20 C. THE SYSTEMATIC COMPONENT OF THE UNCERTAINTY OF THE VALUES IN THIS REPORT IS BASED ON THE UNCERTAINTY OF THE VALUE FROM THE INTERFEROMETRIC DETERMINATION.

THE BOUNDS FOR THE EFFECTS OF RANDOM ERRORS IN THE INTERCOMPARISONS ARE 3 TIMES THE TOTAL STANDARD DEVIATION OF THE PROCESS. WHEN THE BLOCKS ARE OF DIFFERENT MATERIAL THAN THE STANDARDS, A CORRECTION IS MADE FOR DIFFERENTIAL PENETRATION AND THE UNCERTAINTY VALUE IS INCREASED BY ONE MICRO-INCH.

THE MAGNITUDE OF SYSTEMATIC ERRORS FROM SOURCES OTHER THAN THAT OF THE ACCEPTED VALUES FOR THE REFERENCE ARTIFACTS IS CONSIDERED NEGLIGIBLE AT TEMPERATURES VERY NEARLY 20 C. IT SHOULD BE NOTED THAT THE MAGNITUDE OF THE UNCERTAINTY REFLECTS THE PERFORMANCE OF THE MEASUREMENT PROCESS USED TO ESTABLISH THESE REFERENCE VALUES.

THE UNCERTAINTY IN ASSIGNED VALUE CONTAINED IN THIS REPORT BECOMES A SYSTEMATIC ERROR FOR THE LENGTH MEASUREMENTS OF THE USER. IN THE ABSENCE OF OTHER SIGNIFICANT SYSTEMATIC EFFECTS IN THE USER'S MEASUREMENT PROCESS (A CONDITION WHICH MUST BE DEMONSTRATED) THE UNCERTAINTY OF THE VALUE ASSIGNED BY THE USER IS AN APPROPRIATE COMBINATION OF THE SYSTEMATIC ERROR IN THE STANDARD AND THE RANDOM COMPONENT ASSOCIATED WITH HIS PROCESS. IF THE MEASUREMENT PROCESSES ARE IN CONTROL AND APPROPRIATE UNCERTAINTIES ARE ASSIGNED, THE VALUES PRODUCED BY DIFFERENT MEASUREMENT FACILITIES WILL HAVE OVERLAPPING UNCERTAINTY BANDS. ONE CANNOT DISCUSS DIFFERENCES IN VALUES FOR THE SAME OBJECT OBTAINED BY DIFFERENT FACILITIES WITH ANY DEGREE OF SERIOUSNESS UNLESS EACH VALUE IS ACCOMPANIED BY A REALISTIC UNCERTAINTY STATEMENT.

FIGURE

```
        TTTTTTTTTT               TTTTTTTTTTTT
       T      TR.               T  SIZE  TR.
      T SIZE TR .              T   ...  TR .
     T      TR  .             T   .  .X TR .
    T   X  TR   .            T    ... TR   .
   T      TR   .            T        TR    .
  T      TR   .            TTTTTTTTTTTR    .
 T      TR   .             .          .    .
TTTTTTTTTTR  .             .          .   .
.        .  .              .          .  .
.        . .               .          . .
.        ..                .          . .
..........                 .          ..
                           ............


        TTTTTTTTTT               TTTTTTTTTTTT
       T      TR.               T         T.
      T      TR .              T   ...   T .
     T      TR  .             T   .  .  T  .
    T   X  TR E .            T    ...  T   .
   T      TR Z .            T    X    T    .
  T      TR I .            TTTTTTTTTTTT    .
 T      TR S .             .RRRRRRRRRR.    .
TTTTTTTTTT   .             .          .   .
.        .  .              .          .  .
.        . .               .   SIZE   . .
.        ..                .          . .
..........                 .          ..
                           ............
```

THE 'T' REFERS TO TOP CONTACT SURFACE.
THE 'TR' REFERS TO THE REFERENCE EDGE.

THE GAGING POINT :X: IS LOCATED AT THE CENTER OF
RECTANGULAR BLOCKS AND MIDWAY BETWEEN THE HOLE
AND THE REFERENCE EDGE ON HOKE STYLE BLOCKS.

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